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THE JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY

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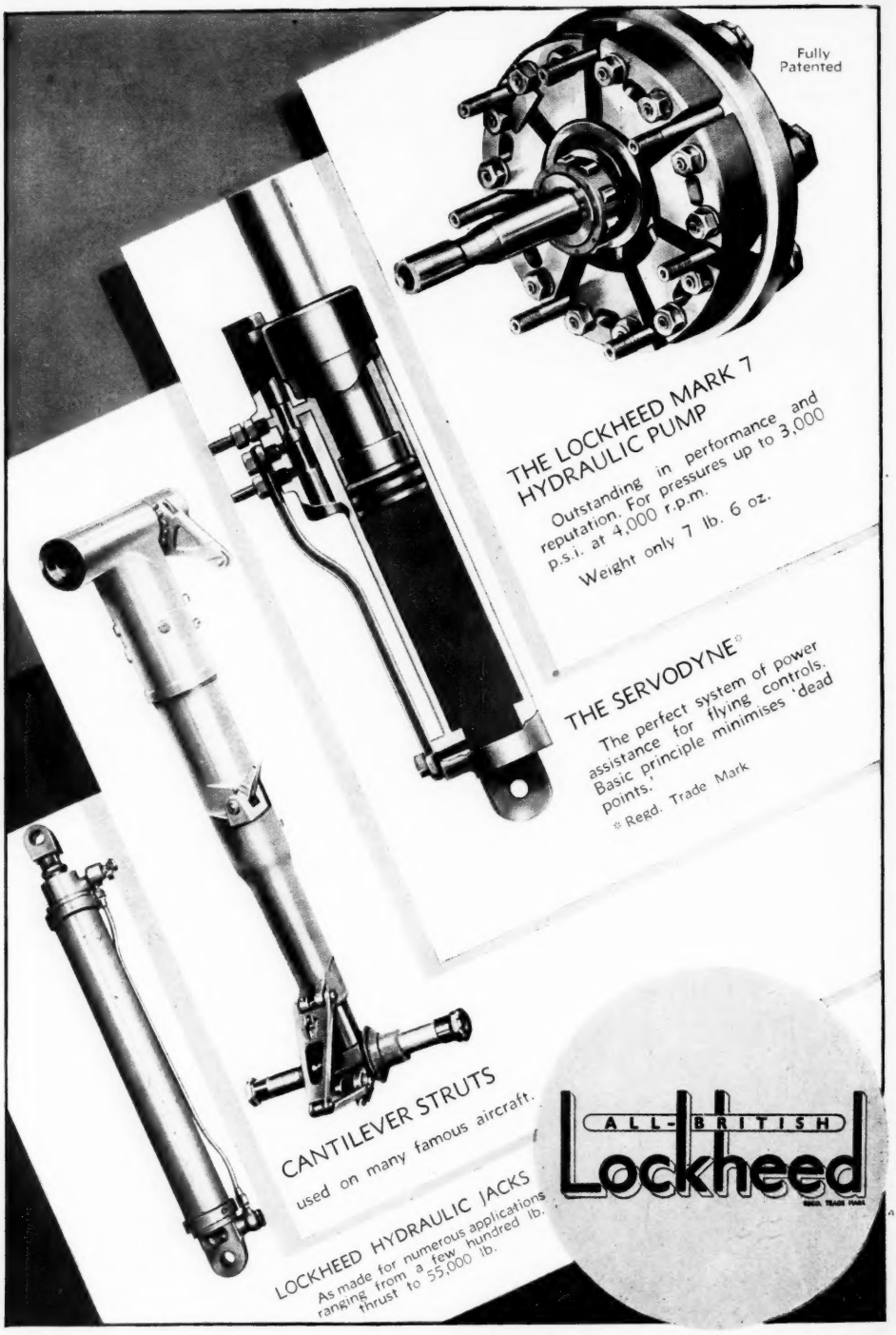
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Secretary's News Letter

APRIL 1952

BRANCH LECTURES are held in many and varied settings; they are held in works canteens, electricity showrooms, town halls, schools and colleges. Pride of place must be given to the Reading Branch who hold their lectures in a picture gallery at the Abbey Gateway where the walls are decorated with historical paintings of Kings and Queens of England. In this atmosphere Mr. A. N. Clifton, Vickers-Armstrongs Ltd. (Supermarines), gave his lecture—"Problems of Transonic Flight"—to an attentive and appreciative audience on Wednesday 6th February 1952.

Before the lecture Mr. D. C. Smith and I visited Handley-Page (Reading) and were shown over the former Miles factory by Mr. E. W. Gray, the Chief Designer. The "Marathon" was the source of interest. Let us hope that it lives up to its name and proves a stayer.

We were entertained at the George Hotel prior to the Lecture; among those present were Mr. E. W. Gray, Mr. A. N. Clifton, Mr. E. L. Pearson, the Branch Secretary, and other members of the Committee.

On Thursday, 21st February, a Main Society Lecture was given at Hull to the Brough Branch by Mr. K. G. Hancock and Mr. P. Person. The title was "Power Steering for Aircraft." Mr. G. H. Dowty, a Vice-President of the Society, acted as Chairman. The discussion almost developed into an argument, especially over the question of "shimmy."

After the lecture we crossed over to the Station Hotel where dinner had been arranged for a considerable party. Mr. G. E. Petty, the Branch Chairman, said a few words, and Mr. Dowty suitably replied, regretting the absence, through illness, of Mr. Robert Blackburn, President of the Branch.

During my stay at Hull I took the opportunity of visiting the Blackburn & General Aircraft Co. Ltd. and learned some of the history of the firm from Mr. F. A. Wilkinson, the Honorary Secretary of the Branch. I was informed of the drawing table, which had been originally a bar, from which the accessories had not even been removed. It makes a good story !

I also visited Hull Technical College, which surprisingly was well-equipped aerodynamically, for its size.

At Blackburns, for the first time in his new surroundings, we met Mr. N. E. Rowe and we wished him well.

The Prize Distribution of the Royal Aircraft Establishment Technical College was held at Farnborough on the 5th March, when Mr. Kenneth Pickthorn, D.Litt., M.P., Parliamentary Secretary to the Minister of Education, presented the prizes. Dr. Pickthorn's speech was much appreciated by a laughing audience of students and guests. Sir Frederick Handley-Page, the Chairman of the Board of Governors, also quipped a few reminiscent stories. The Principal, Mr. R. D. Peggs, is to be congratulated on a most successful academic year.

It seemed to me, accustomed to more frugal prize-givings, that this Technical College had very many prizes to present. The wise had gifts as well as the more customary words, and this occasion merits the coining of a new tag—"don. sap.," gifts to the wise.

To the disappointment of everyone the President was not sufficiently recovered to attend the Fifth Bleriot Lecture in Paris on the 12th March, but Air Commodore F. R. Banks, C.B.E., made an able substitute.

I must confess I liked the technique of Mr. Knowler in reading a few sentences in French and then passing over his lecture to be read by M. Benoit.

Mr. Knowler, Fifth Bleriot Lecturer, and M. Jarry, President of A.F.I.T.A., at Versailles.

The Lecture was followed by a Cocktail Party and later a Dinner. The Association Française des Ingénieurs et Téchniciens de l'Aéronautique made this the occasion of their Annual Dinner, and there was a good attendance. About 40 British representatives were present.

We had speeches from M. Jules Jarry, the President of A.F.I.T.A., Air Commodore Banks, M. Hereil, President of the Chambre Syndicate des Industries Aéronautiques (the French S.B.A.C.) and M. Pierre Montel, Secretary of State for Air. Had they been Greek to me I might have understood them better.

On the Thursday afternoon at the invitation of M. Roy, the Director, we visited the Office Nationale d'Etudes et de Recherches Aéronautiques (O.N.E.R.A.). It was an interesting visit; there was one research technique for plotting flow patterns with Teledeltos paper which I had never seen before.

An eminent member of the Society, on his return from Paris, told me that he didn't really go to Paris for the Lecture. When I smiled, he hastened to add, "I went to see members of the British firms, for then they have leisure to talk." "Qui s'excuse, s'accuse."

Mr. George Edwards, Chief Designer of Vickers-Armstrongs Ltd., Weybridge, gave a lecture on the "Viscount" to the Southampton Branch on Wednesday 19th March. Professor Richards, the Branch Chairman, was in the Chair. Mr. Edwards was in good fettle, and kept his listeners attentive, lest they miss the next pungent remark. It was an enjoyable lecture; the adjective is, I think, the appropriate one. After the lecture, which was held in the College at Southampton, Mr. T. Gilbertson, Director and General Manager of Folland Aircraft Ltd., gave a small dinner; among those present were Mr. George Edwards, Professor E. J. Richards, Mr. H. C. Smith, Secretary of the Branch, Mr. J. Smith, Chief Designer of Vickers-Armstrongs (Supermarine), Mr. A. N. Clifton, Mr. F. H. Pollicutt, Chief Designer of Follands and Wing Commander D. B. Smith, of Follands.

The Halton Branch have decided to institute a Laurence Dean Memorial Trophy for the most oustanding lecture by an Air Apprentice member of the Branch. It will be awarded annually. The subscription list has been opened, and donations should be sent to the Branch Secretary at the R.A.F. Station, Halton.

A. M. Ballantyne

Secretary

NOTICES

ANNUAL GENERAL MEETING, 7th MAY 1952

Notice is hereby given that the Annual General Meeting of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, will be held on Wednesday 7th May 1952 at 6 p.m. in the offices of the Society, 4 Hamilton Place, London, W.1.

AGENDA

1. To read the Notice convening the Meeting.
2. To receive and deliberate upon the Report of the Council on the state of the Society and the Balance Sheets and Income and Expenditure Accounts of The Royal Aeronautical Society and Aeronautical Trusts Limited for the year ended 31st December 1951.
3. To receive the names of those elected to Council for the years 1952-1955.
4. To announce the names of Fellows elected by the Council in accordance with By-Law 4.
5. To elect the Auditors for the year 1952.
6. Any other business.

By Order of the Council,

A. M. BALLANTYNE,
Secretary.

2nd April 1952.

NOTE.—In accordance with the By-Laws any member whose subscription has not been paid before the first day of April is not entitled to vote.

Light refreshments will be served after the meeting.

CONTENTS OF APRIL JOURNAL

Structural Problems of Advanced Aircraft, H. H. Gardner, B.Sc., F.R.Ae.S.
A Note on the Vortex Patterns in the Boundary Layer Flow of a Swept-Back Wing, Joseph Black.
A Method of Estimating the Loading on an Elastic Airframe, B. A. Hunn, B.Sc.
87th Annual Report of the Council.
Reviews. Additions to the Library. Correspondence.

(An annual sum of £250 is available for premium awards for papers published in the Journal. These premium awards are usually 15 guineas each. Members and non-members of the Society are invited to submit papers on any aspect of aeronautics.)

THE AERONAUTICAL QUARTERLY—VOLUME III, PART IV

The Aeronautical Quarterly, Volume III, Part IV, February, is now available from the offices of the Society, at 7s. 6d. a copy to members (7s. 9d. post paid) and 10s. (10s. 3d. post paid) to non-members.

The contents of Part IV are:—

The Automatic Control of an Aeroplane in the Landing Approach onto an Aircraft Carrier ... ... J. B. Helliwell
Supersonic Flow over Thin Symmetrical Wings with given Surface Pressure Distribution ... ... F. A. Goldsworthy
Longitudinal Waves in Beams ... ... R. E. D. Bishop
A Note on Supersonic Wing Integral Equations in Unsteady Flow ... ... John W. Miles
The Theory of Aerofoils in Unsteady Motion ... ... J. R. M. Radok
Index to Volume IV.

Copies of Volume III, Parts I, II and III, are still available from the offices of the Society at 7s. 9d. a copy to members of the Society, post paid, or 10s. 3d. to non-members, post paid.

GARDEN PARTY, 15th JUNE 1952

The Society's Annual Garden Party will be held on Sunday 15th June 1952 at White Waltham Aerodrome, near Maidenhead, by kind permission of the Ministry of Civil Aviation and the Fairey Aviation Co. Ltd. An application form will be sent to all home members.

An attractive flying programme will be arranged including some gliding, helicopters and light machines from this country and abroad.

MEMBERS' NEW APPOINTMENTS

E. W. BRUCE-HILL (*Associate*), formerly with the Experimental Design Staff of the De Havilland Aircraft Co., is participating in an Antarctic expedition for the next two years. He will be engaged mainly on research and development of polar equipment and his address will be c/o The Postmaster, Port Stanley, Falkland Islands.

W. E. CLARK (*Associate*) has retired from B.O.A.C. after 32 years' service. He was Engineering Manager at London Airport.

F. G. LYNAM (*Fellow*), formerly Technical Director, has been appointed General Manager of the Airscrew Co. Ltd., of Weybridge.

A. L. RUCKLEY (*Associate Fellow*), formerly of Folland Aircraft Ltd., has taken up an appointment as Experimental Officer in the Chief Scientist's Division of the Ministry of Fuel and Power.

H. B. SQUIRE (*Fellow*) has been appointed to the Zaharoff Chair of Aviation at the Imperial College of Science and Technology, London, in succession to Professor A. A. Hall.

ASSOCIATE FELLOWSHIP EXAMINATION

The next Associate Fellowship Examination will be held in the offices of the Society on 26th, 27th and 28th May 1952. All candidates who have entered for the Examination will be informed individually of the detailed arrangements.

FORTIETH WILBUR WRIGHT MEMORIAL LECTURE 1952

The Fortieth Wilbur Wright Memorial Lecture will be given by Sir Harry M. Garner, K.B.E., C.B., M.A., F.R.Ae.S., Chief Scientist to the Ministry of Supply, on 29th May 1952 on "Prophecy and Achievement in Aeronautics." The Lecture will be held at the Institution of Civil Engineers, Great George Street, S.W.1, at 6 p.m.

EIGHTH BRITISH COMMONWEALTH AND EMPIRE LECTURE 1952

The Eighth British Commonwealth and Empire Lecture will be given in October 1952 by Mr. R. E. Hardingham, O.B.E., *Fellow*, Secretary and Chief Executive of the Air Registration Board. The exact date and title of the lecture will be announced later.

AIRFIELD PAVEMENTS FOR JET AIRCRAFT

A symposium on "Airfield Pavements for Jet Aircraft," sponsored by the U.S. Naval Civil Engineering Research and Evaluation Laboratory, is being held on the 17th and 18th April at Port Hueneme, California. Among the bodies taking part in the Symposium are the U.S. Navy, Army and Air Force, the U.S. Civil Aeronautics Administration, Aircraft Manufacturers, the British Joint Services Mission, the Royal Canadian Air Force, the Portland Cement Association, the Asphalt Institute and a number of Universities. Among the papers given will be one on the "British Approach to the Problem" by a representative of the Air Ministry.

Anyone interested in reports of this Symposium should write to J. A. Bishop, Director, Soils and Pavements Division, U.S. Naval Civil Engineering Research and Evaluation Laboratory, Port Hueneme, California.

DIARY

LONDON.

April 22nd.

GRADUATES AND STUDENTS SECTION—Servomechanisms: Their Principles and Uses, F. H. Borlace. 4 Hamilton Place, W.1. 7.30 p.m.

SPECIAL LECTURES

April 24th.

MAIN SOCIETY LECTURE AT LUTON—Fuel Systems for Turbine-Engined Aircraft, J. E. Walker. The High School for Girls, Alexandra Road, Luton. 6.30 p.m.

May 29th.

40TH WILBUR WRIGHT MEMORIAL LECTURE—Prophecy and Achievement in Aeronautics, Sir Harry M. Garner, Institution of Civil Engineers, Great George St., London. 6 p.m.

BRANCHES.

April 15th.

Belfast—Annual General Meeting. College of Technology. 7 p.m.

April 16th.

Hatfield—Annual General Meeting. de Havilland Aircraft Co. Ltd. 6.15 p.m.
Weybridge—Modern Aircraft, G. R. Edwards. Vickers-Armstrongs Ltd. 6 p.m.

April 17th.

Gloucester and Cheltenham—Annual General Meeting and Film Show. St. Mary's College, Cheltenham. 7.30 p.m.

April 22nd.

Bristol—Annual General Meeting and Film Show. Filton House. 6 p.m.

April 23rd.

Brough—The Testing of Experimental Aircraft, Lieut. R. M. Reynolds, D.S.C., A.F.C., R.N. Electricity Showrooms, Hull. 7.30 p.m.
Coventry—Annual General Meeting and Films. The Wine Lodge. 7.30 p.m.

April 24th.

Luton—MAIN SOCIETY LECTURE—Fuel Systems for Turbine-Engined Aircraft, J. E. Walker. The High School for Girls, Alexandra Road. 6.30 p.m.

April 26th.

Hatfield—Annual Dinner.

April 28th.

Halton—Film Evening. Branch Headquarters. 6.45 p.m.

May 5th.

Derby—Wind Tunnels, F. B. Greatrex. Rolls-Royce Welfare Hall. 6.15 p.m.
Halton—Branch Night. Branch Headquarters. 6.45 p.m.

May 7th.

Luton—History of Helicopter Propulsion Systems, L. R. Hayward. The George Hotel.

May 12th.

Halton—Electric Power in Aircraft, V. H. Higgs. Branch Headquarters. 6.45 p.m.

May 14th.

Chester—Annual General Meeting. Grosvenor Hotel. 7.30 p.m.
Weybridge—Annual General Meeting. Vickers-Armstrongs Ltd. 6 p.m.

May 19th.

Halton—Films. Branch Headquarters. 6.45 p.m.

May 23rd.

Birmingham—Annual General Meeting. White Horse Hotel. 7 p.m.

HONOURS AWARDED TO MEMBERS

Professor A. G. Pugsley, *Fellow*, and Dr. D. M. Smith, *Fellow*, have recently been awarded the honour of Fellow of the Royal Society.

NORTHERN HEIGHTS MODEL FLYING CLUB, GALA DAY

Members of the Society have been invited to attend the Gala Day of the Northern Heights Model Flying Club which will take place on Sunday 29th June 1952 at the Hawker Aircraft Company's aerodrome, Langley Aerodrome, Bucks., when the Queen's Cup Competition will be held.

ELLIOTT MEMORIAL PRIZE

The Elliott Memorial Prize has been awarded to Sergeant Aircraft Apprentice G. T. Bodggood of the June 1949 Entry at R.A.F., Halton, who has obtained the highest marks in the General Studies Examination. The prize will be presented at the Graduation Prize Giving on 27th May 1952.

ELECTIONS

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

Dennis Ashton (*from Graduate*)
Ralph Elwin Roy Broomfield (*from Graduate*)
Denis William Bryer (*from Graduate*)
Charles Head Bulmer
George William Burley (*from Associate*)
Guy Bransby Charter (*from Graduate*)
James Arthur Collopy
Raymond Vincent Cudmore (*from Graduate*)
James Dougal Harris (*from Graduate*)
Frank Henry Higgs (*from Associate*)
John Arthur Godfrey Holmes (*from Graduate*)
Frederick Henry Hooke
Samuel Ronald Hughes
Maurice Jackson (*from Graduate*)
Flavin Earl Loudy
Romney Hollins Lowry (*from Graduate*)
Ronald James Maccallum (*from Associate*)
Spencer Davidson Meston (*from Graduate*)
Wilfred Thomas Morgan (*from Associate*)
Andrew John Stephen Morris (*ex-Associate*)
James Ernest Raddings
Stanley Grapes Shand
Lionel John Augustus Sice
Norman Vinicome Slatter
Neil Hickman Stewart (*from Graduate*)
Christopher Sutcliffe (*from Graduate*)
Baboo Ram Teree
Francis Todd (*from Graduate*)
John William Jameson Truran (*from Associate*)
Alan John Wangford (*from Graduate*)
Henry Romaine Watson
Albert Edward White (*from Graduate*)

Associates

Charles Dudley Anderson (*from Companion*)
Alec Lionel Boughton
William Coppen

Frederick Eric Draycott
Alfred Joseph Gooding
Frederick Gray
Horace John William Holding
Mary Howard
John Owen Isaacs
Michael Iliesco-Constantyne
David Michael (*from Student*)
John David Perkins
Donald Portas
Frank Milnes Prickett
William McDonald Seymour
Charles Ernest Stafford
Kenneth Stuart-Smith (*from Companion*)

Graduates

Benjamin James Allen
Thomas Frank Arlotte
John Barton Armstrong
Roger Frederick Back (*from Student*)
Ronald Frank Bailey (*from Student*)
Derek Bollworthy (*from Student*)
Peter Alfred Thomas Christopher (*from Student*)
John Egon Forbat
Arend Frans Haye
Kenneth Herbert Haywood
Patrick Joseph McMahon (*from Student*)
Donald Pierrepont (*from Student*)
John Dennis Poole (*from Student*)
John Fleming Purdy
David Worth Wall (*from Student*)
Sydney George Wheeler
Sydney William White

Students

Eric Stanley Barwick
Michael James Clarke
Reginald Francis Maurice Drake
David Alan Drane
Vincent Michael Reginald Fitzgerald
Leslie James Geering
Alan Raymond Guy
John Bevil Hilton
Brian Ross Langevad
Leo Francis McCarty
John Masefield
Theodor Opatowski
Colin Charles Parker
Graham Frederick Poyser
Raymond John Stalker
David Clay Sugden
Christopher John Hamshaw Thomas
Redvers Cecil Warren

Companions

Mario Bernardi
Clause Evelyn Bowden
Thomas Hamilton Trimble

ACKNOWLEDGMENTS

The Council acknowledge with grateful thanks the return of back numbers of the JOURNAL from Squadron Leader R. G. Simmons, *Associate Fellow*; B. Jablonsky, Esq., *Associate Fellow*; J. D. Campbell, Esq., *Associate Fellow*; F. Walsh, Esq., *Associate Fellow*; Wing Commander D. M. Egan, *Associate*; and H. W. Dunn, Esq., *Associate Fellow*.

JOURNAL BINDING

Members are reminded that a special leaflet on JOURNAL binding for the 1951 Volume and for 1952 was enclosed with the December JOURNAL. Copies of this leaflet are available on application to the Society.

CHANGES OF ADDRESS

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).

Grade of membership.

New address (in block letters).

Old address.

Changes of address must be received b*efore the 15th of the month* in order to be effective for the JOURNAL for the following month.

Structural Problems of Advanced Aircraft

by

H. H. GARDNER, B.Sc., F.R.Ae.S.

The 835th Lecture to be given before the Royal Aeronautical Society was delivered at the Institution of Civil Engineers, Great George Street, London, S.W.1, on 29th November 1951. Mr. G. R. Edwards, M.B.E., F.R.Ae.S., Vice-President of the Society, presided and introduced the Lecturer, Mr. H. H. Gardner, B.Sc., F.R.Ae.S., Assistant Chief Designer, Vickers-Armstrongs Ltd., Weybridge.

1. INTRODUCTION

In assessing the true extent of technical development of any kind the mind naturally seeks a standard for comparison. When considering structural development the standard chosen could well be that associated with the end of the Second World War. In preparing this paper, therefore, it appeared profitable, and certainly it has been extremely interesting for the author, to review the structure of some of the aircraft then in quantity production. This course has been pursued in this brief review and it is hoped that by showing a few illustrations of some of the more typical structures of that period a truer impression of the advances made in the past few years may be obtained.

Any discussion of structural problems, sooner or later, will reach the basic problem containing the eternal triangle of strength, stiffness and structural weight. The satisfactory provision of the first two for a minimum expenditure of the third will always form the hard core of structural development.

The most important difficulties, however, are generally those which require an immediate solution. If, therefore, in this paper, the author has devoted too much attention to the immediate problems, it is in the belief that, when compared with more distant tasks, they are at present of greater interest to a larger number of structural workers and will provide a more topical subject for general discussion.

2. STRUCTURAL DEVELOPMENT

The history of structural development can be traced over the past 20 years from the biplane to present day high-speed aircraft as a sequence of related steps. For the purposes of this paper it is convenient to divide this period, beginning at the end of the First World War, into four stages of development. These, from the structural viewpoint, can be considered quite logically under the four headings of

1. Biplane structure
2. Monoplane development
3. Civil development
4. Recent military development.

These four stages of structural development begin with the biplane era with its skeletal structures in which the thinnest fabric covering was used on both wooden and metal space frames of spars, ribs and interplane struts. Monoplane development followed, producing problems for the structural designer more varied, of a much wider range, and certainly much more numerous than ever before. It was a period of intense development when design was both fertile and fluid. Before the end of the Second World War the need for civil transport aircraft was foreseen, introducing a period of general retrenchment in which structures, which had been produced in quantity, were refined and adapted. The fourth stage covering the development of high-speed military aircraft of the present day was hastened for political reasons. The bulk of this paper is devoted to the problems which are of some importance in this period, but in order to bring these more clearly into focus it is thought that mention should be made of the important problems which engaged the attention of structural designers in the earlier stages.

2.1. Stage 1. Biplane structures

In this period structures were complete space frames and much attention was paid to Castigliano's theorem regarding the number of members and joints. The three chief problems on which the majority of structural work was done were

(*i*) The design of mainplane spars, particularly when the spruce sections were replaced by light alloy and steel.

(*ii*) The design of struts in which the interplane strut was given pride of place.

(*iii*) The design of main joints.

The problem (*iii*) still is, and always will be, one which requires a great deal of attention by all structural engineers. Those most successful in the art will admit that however good the engineering training of the designer, the problem has as much to do with art as with science.

2.2. Stage 2. Monoplane development

This stage covers a period of steady development which began when the introduction of landing flaps made practicable the higher wing loadings necessary for the successful design of the monoplane. It is interesting to recall that the small civil aeroplane was the first to break away from current practice, in that a modern monoplane with a retractable undercarriage was produced as a civil aeroplane before any serious attempt had been made to introduce into the R.A.F. a military aeroplane with these features. The introduction of the monoplane intensified research on the wing thickness/chord ratio and on wing taper. Efforts were made to obtain the maximum value of thickness/chord ratio which could be accepted without loss of performance; thus wing sections with a thickness/chord ratio of around 20 per cent became the rule rather than the exception, a change which represented rather more than a 30 per cent increase on the thickness/chord ratio used in the average biplane. This particular point is noted, because in the author's recollection it is one of the outstanding occasions where a structural requirement led, rather than followed, the aerodynamic design.

Figure 1 illustrates wing thickness/chord ratios for both root sections and tip sections for aircraft which have been developed since the middle of the second period and in many cases are still in use today. It will be seen that the thickness/chord ratio at the root varies from some 22 per cent. down to 12 per cent., while the thickness/chord ratio at the tip varies with different types of aircraft and

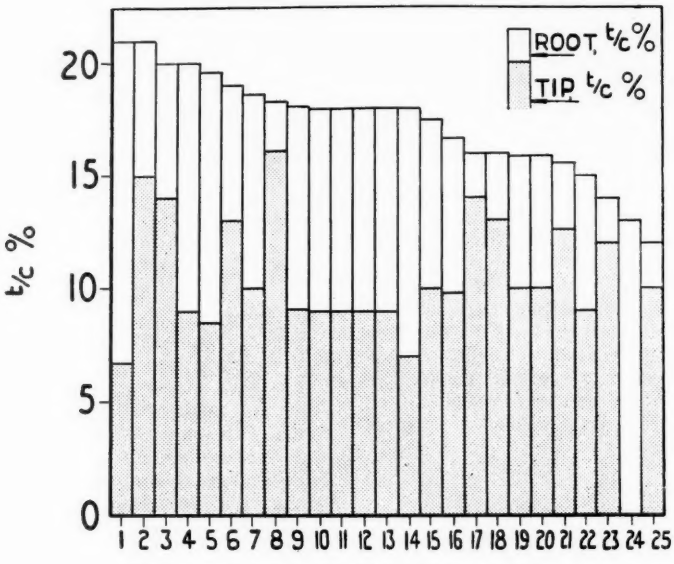


Fig. 1. Wing root and tip thickness/chord ratios.

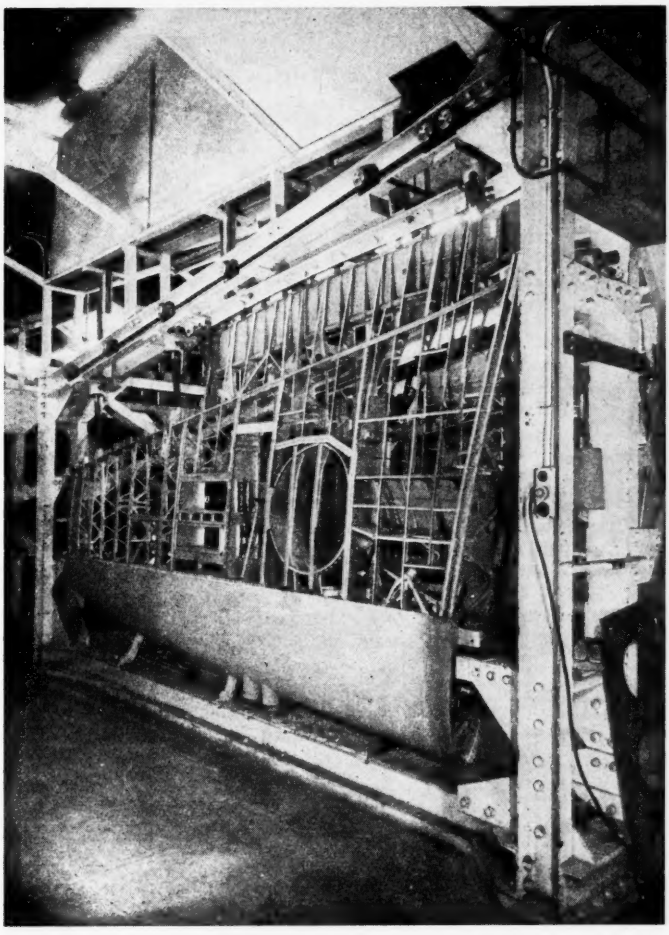


Fig. 2.

follows no obvious trend. The continued reduction of thickness/chord ratio, however, is clearly one of the major problems which will have to be faced in aircraft of the future. How far this reduction can be tolerated structurally is a matter for conjecture, but certainly in high-speed supersonic aircraft thickness/chord ratios of 2 per cent. are now being freely discussed.

With the general adoption of the monoplane for military aircraft the Industry settled down into well-defined channels, in that certain firms concentrated on certain types of aircraft; the fighter, the medium aircraft of the fighter-bomber class and the heavy transports or bombers became associated with certain names. This was even more pronounced when the rearmament programme in the late 1930's was introduced.

This programme, for the first time, made quantity production of metal aircraft one of the important factors in aircraft design. A great deal has been said and written of the wisdom of the planners, at the various Ministries during this period, in producing the right specifications at the right time. The planning and production engineers who made possible the quantity production of the aircraft illustrated in this paper deserve a similar tribute.

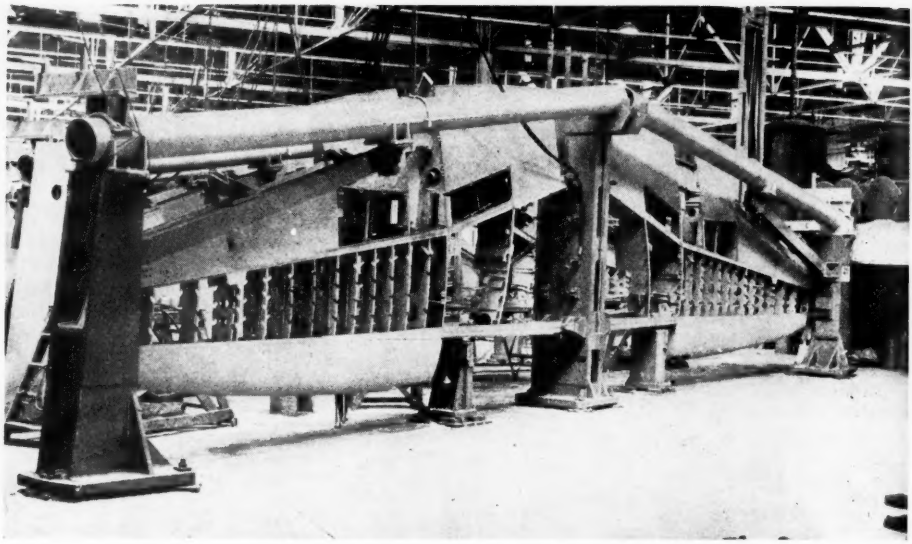
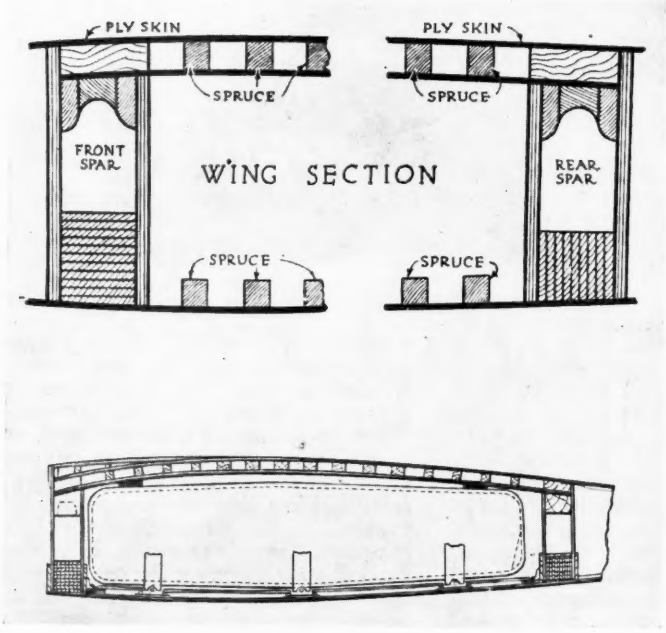


Fig. 3.



Section of mainplane outboard.

Note laminated construction of box spar flanges.

Section of mainplane showing the arrangements and mounting of the petrol tanks between the main spars. The bottom surface of the wing is detachable at this point and consists of balsa wood sandwiched between two sheets of plywood, thus forming a door. The door acts as an integral part of the wing structure, the stresses being transmitted via bolts through the fabric-reinforced plastic borders of the door.

Note English ash reinforcements to bottom flange of the box spars.

Fig. 4.

The accelerated development of this period, due to the initial incentive of a world war, quickly followed by the lessons of actual combat, produced many famous aircraft; thus the end of this war may be regarded as a stage in structural development which is worthy of some consideration before proceeding to the more difficult problems which have arisen in the past six years.

Through the generosity of the firms responsible for the design and construction of those aircraft in quantity production in the mid 40's it has been possible to show some of the main structural components of aircraft produced in large numbers. It is impossible in this brief review to do justice to the structural development of all the aircraft produced at this time, but it is hoped that the illustrations which have been prepared show the varying types of structure in use during this period, and illustrate the structural development up to this time.

Figure 2 shows a wing which will be recognised as that of an almost legendary fighter produced by Supermarines. It illustrates a wing with a D-section spar which was developed and produced for many years, with the consequence that it is so well known that no further description is necessary.

Figure 3 shows the wing of another fighter aircraft produced by the Hawker Company in which is seen a two-spar construction merging to monocoque in the outboard section. This wing is one of several having similar features, which suggests a tendency towards standardisation.

Structures of this type have been employed in many successful aircraft.

Figure 4 shows an entirely different type of construction, a unique design comprising a plywood sandwich produced by the de Havilland Company. Spruce spanwise stringers were attached to inner and outer plywood skins. The front and rear spars were of laminated construction and deal mainly with the tension stresses as the top surface, being continuous throughout, reacts compression stresses as a sandwich skin or shell. The general pattern of box outer

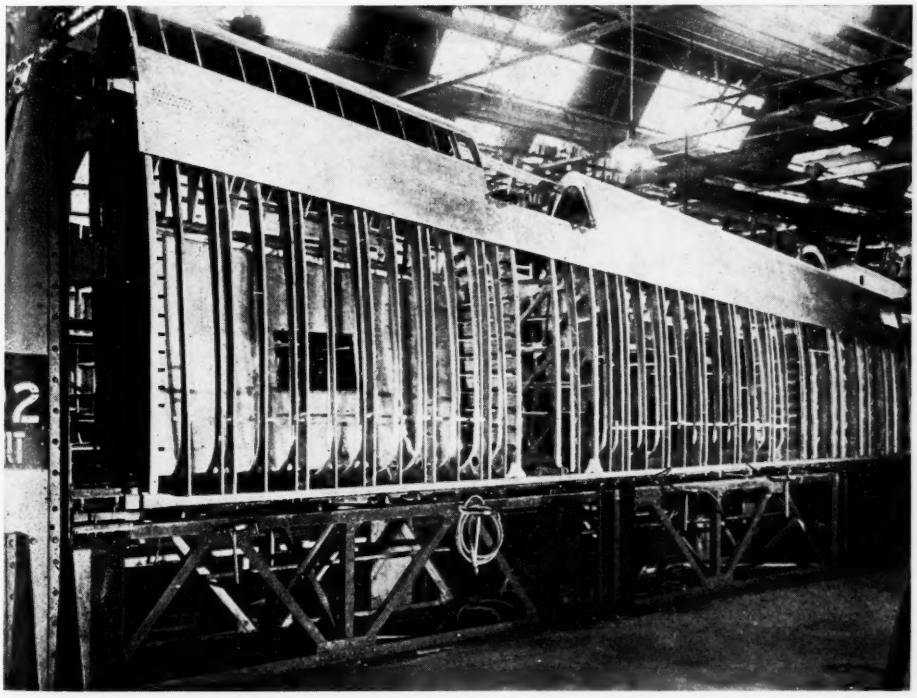


Fig. 5.

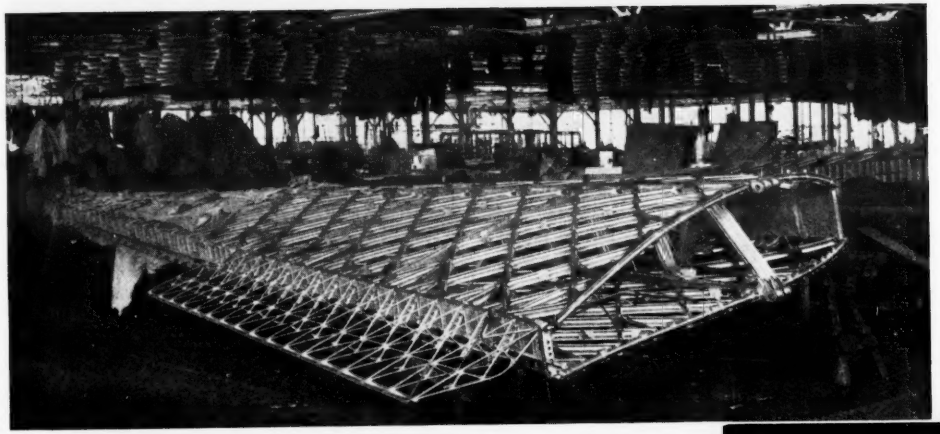


Fig. 6.
(Left)

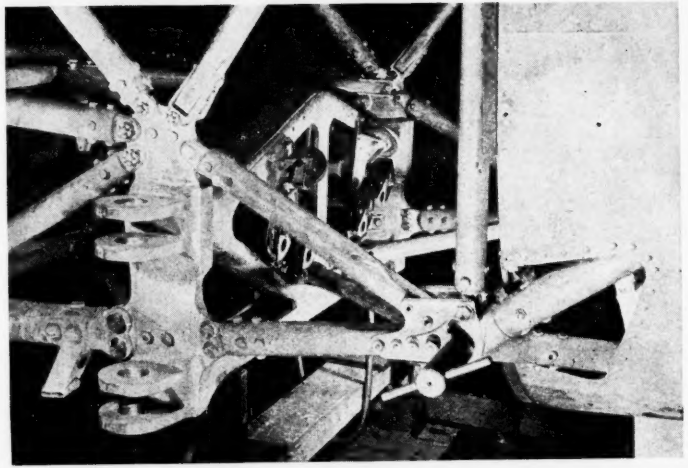


Fig. 8.

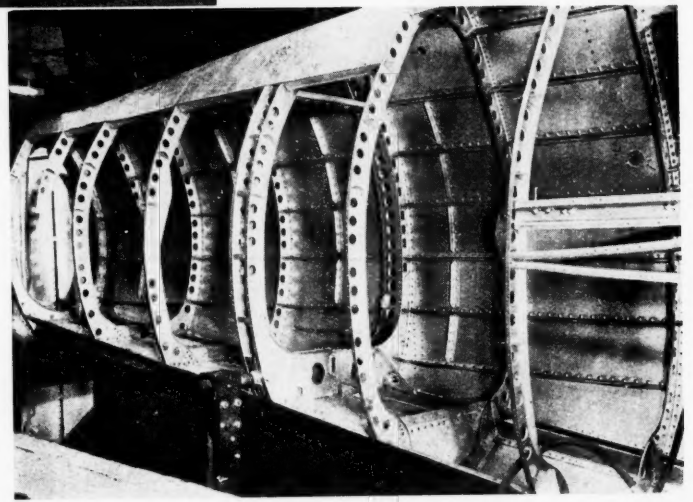
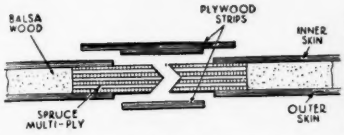
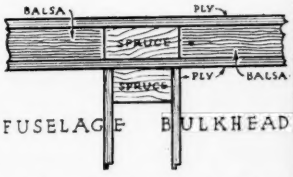


Fig. 7.

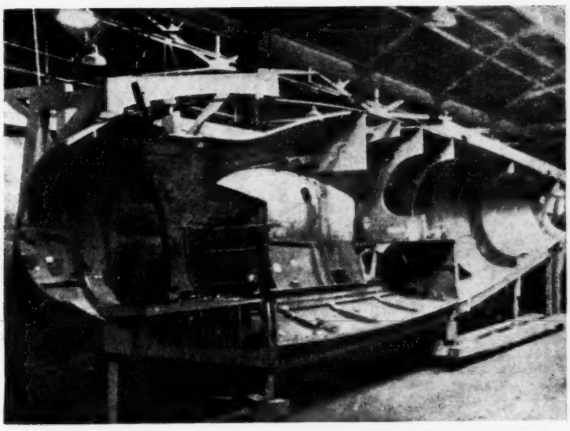


Details of the joint between the fuselage halves.



Typical section through the balsa plywood sandwich at one of the bulkhead sections.

Fig. 7.



The fuselage is constructed in two longitudinal halves complete with bulkheads and internal structural members.

Fig. 9.

Fig. 8.

wing merging to a spar inner plane with its diffusion problems is again seen.

Figure 5 shows the inner wing of a four-engined heavy bomber, illustrating the two-spar construction which is an almost universal solution when the space between is required for fuel tanks, engines or similar purposes. The warren girder type of engine rib is a feature of this wing, so are the formers used in the tank bay.

This form of construction is typical of many wing structures of heavy aircraft in current use during the period under review.

Figure 6 shows another wing of unique construction: the geodetic wing of the Wellington bomber. A single main spar was used with two subsidiary spars or edge members, the inner forces and torque being reacted by the geodetic channels. This was a fascinating wing from the stressman's point of view, and as a warplane the punishment it could absorb and the ease with which it could be repaired made it outstanding.

Figure 7 shows the Spitfire fuselage which illustrates an earlier form of stressed skin construction. This fuselage had longerons where connected to the centre section. The skin was attached to these and to the widely spaced frames.

Figure 8 shows the fuselage centre section of a Hawker fighter and is an example of the continued use of previously successful detail design.

Figure 9 shows an entirely different type of construction, unique among the aircraft structures of this period, the de Havilland construction where a plywood sandwich with a balsa filling was used successfully to produce one of the most successful fighter-bombers of the later war years.

Figure 10 is an example of fuselage construction which was widely adopted in principle towards the end of Stage 2 and is still used extensively up to the present day. Several examples of this type of construction can be shown; no longerons, frames built out to the skin and stringers continuous through frames. Several variations of this type of construction are in existence but, in principle, this form of construction has been generally adopted.

Figure 11 shows a production line of geodetic fuselages still in service today in spite of fabric covering. In this structure the main end loads were carried by four tubular longerons, the geodetic bracing carrying the shear and torque loads, while at the same time eliminating the need for all frames except at main spar attachments.

The next two figures are shown with some trepidation as they illustrate large castings.

Figure 12 shows an aluminium alloy casting (D.T.D.300) which was used to carry both the engine nacelle and undercarriage loads of an Avro four-engined heavy bomber. It

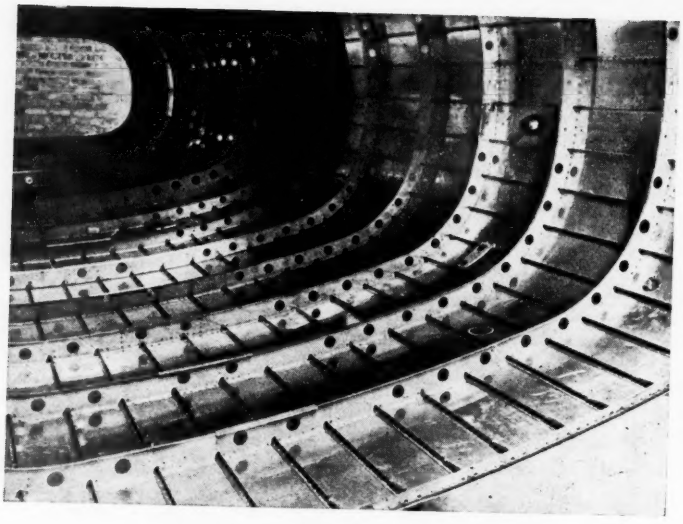


Fig. 10.

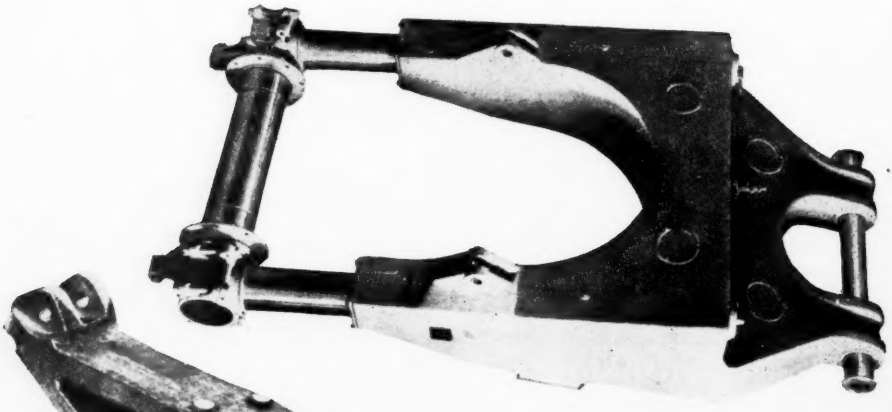


Fig. 13 (above).

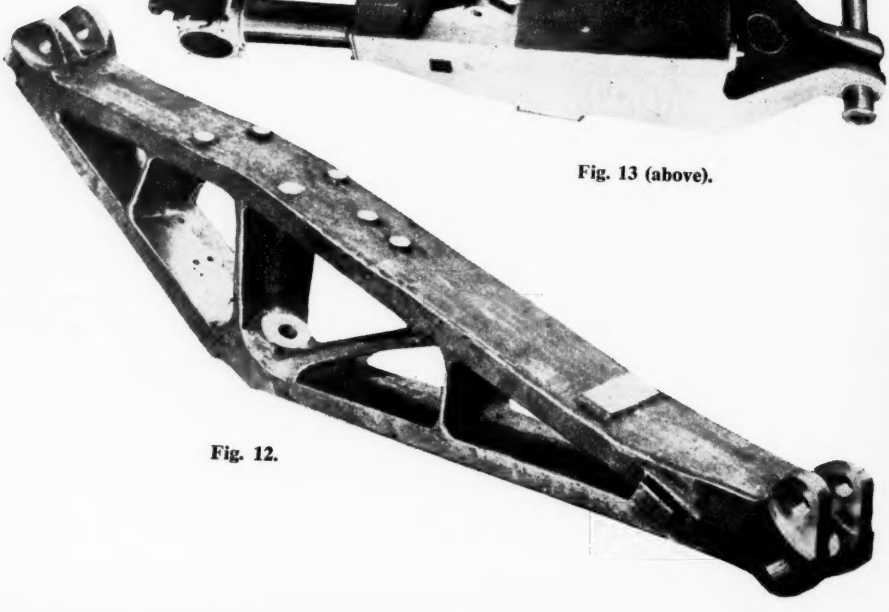


Fig. 12.

was produced in large numbers and retained for several variants of the first aircraft.

Figure 13 shows a magnesium alloy casting (D.T.D.59A) produced by British Messier for the undercarriage of a large Handley Page bomber. This casting in its final form weighed over 200 lb.

It is a matter of interest to reflect that these two particular castings were outstanding some few years ago, and were successful structural components, yet in the Industry there is no general move to continue the use of very large castings. Is this because of the difficulty of meeting requirements, or the difficulty, or cost, of producing sound castings of this size? In view of some of the difficulties recently experienced with large forgings this becomes a matter of interesting speculation.

To close this review of solutions to structural problems of this period, it is considered that the following points should be emphasised:

1. No optimum design has emerged.
2. The continuation of previously successful detail design was a general practice.
3. Stressed skin construction, now generally regarded as conventional, was gradually becoming predominant — particularly when a more complete review of all aircraft in production at this time is made.
4. Where possible wing spars were being reduced and loads carried in the stiffened wing plating. Thus the box was growing from the old two-spar wing, *i.e.* the approach to the idealised shell or the evolution of the structure outwards was proceeding.

2.3. Stage 3. Civil Development

With the end of hostilities the bomber firms turned to the civil market and each of the bomber aircraft shown was recast into a civil transport. In general a new fuselage was required, the remaining components being retained; quick production was the essential feature of the new projects, with the result that conventional stressed skin designs were adopted and structural problems were mainly of a detailed nature.

The operators' demand for metal covering ended the long run of the fabric covered aeroplane and thus ruled out the unconventional forms of construction. With geodetic construction the change to conventional stressed skin for wings and tail surfaces was introduced without change in principle; in the wings the single spar and edge members were retained and the geodetics and fabric were replaced by skin stiffened with chordwise formers. The change from other forms of

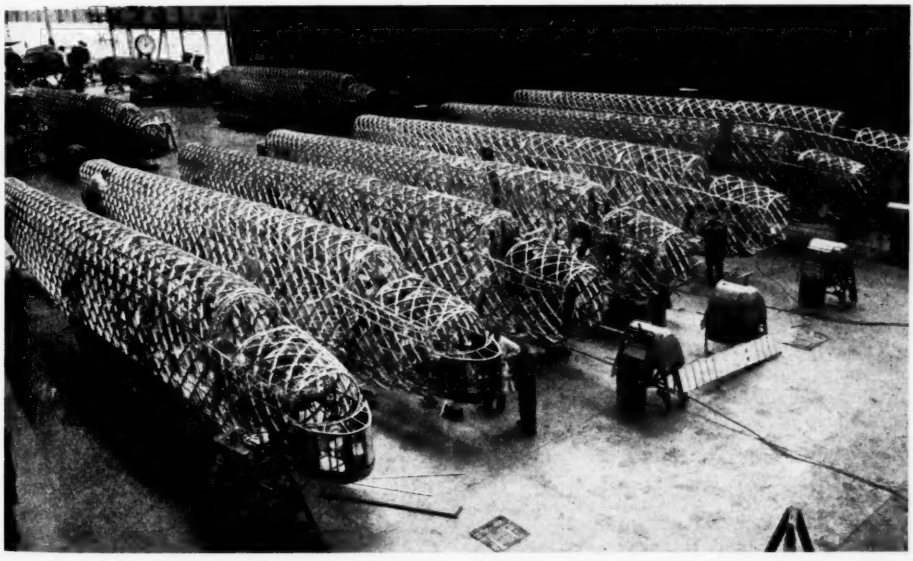


Fig. 11.

unorthodox construction was rather more drastic.

The aircraft produced during this period showed in varying degrees the structural influence of American practice where development had not been subjected to those setbacks which the British Industry had faced during the war years; the result was a saving in time at the expense of some standardisation.

During this stage, and during the latter war years, technicians in the Industry received a great deal of assistance from the Data Sheets produced by the "Stressed Skin" Committee of the Royal Aeronautical Society. The Data Sheets are still in general use throughout the stress offices of the Industry, which owes a debt of gratitude to the Society for the initiation of that work. It is hoped that the continuation of this work by the Structures Committee, in order that the published information may be kept completely up to date, will not prove an impossible task.

While the Industry was busy producing the so-called interim types the Ministry, through the Brabazon Committee, produced a set of specifications to cover future civil demands. These are now known as the Brabazon types and are important in that they will play a decisive part in post-war civil development.

Fortunately for aircraft designers the immediate post-war civil aircraft, with one exception, operated at moderate speeds and heights; 300 knots and 25,000 ft. covered the cruising speeds and altitudes of the majority of these aircraft. Thus the problems of strength and stiffness required at high speed and high altitude did not, in general, become acute; also, at these speeds the gust cases were not significantly more severe than those arising from the manœuvring envelope. Structurally this gradual development was a happy state of affairs as the structural engineer was given more time to progress towards the more advanced aeroplane.

The three years of this period were a time of structural retrenchment, when the problems which became particularly pressing were those of

(*i*) pressurisation,

(*ii*) transparencies and dielectrics,

(*iii*) life of aircraft structures.

These three problems, all of which first reached major proportions during the civil development stage, have continued to remain more or less intractable to the present day. They have therefore won a place among the final problems to be considered.

During this period also, civil requirements were unearthed from the dust of the war years and completely rewritten. It is probably true to say that the work on British Civil Requirements during these years has never before been equalled, and will never be surpassed. There was a natural cause for this activity in the world-wide desire for maximum safety in air transport, which would have produced, in the normal way, new requirements over a long period of years, had it not been for the exceptional impetus given to this work by the Chicago Conference of 1944. The United States Civil Aircraft Authorities had produced a civil code which caused an immediate reaction in Great Britain. International civil aviation was following a powerful American lead with, at the time, apparently no alternative until Britain accepted the challenge.

The newly reorganised Air Registration Board was given the task of rewriting the British Civil Airworthiness Requirements and, with the initial help of a few of the most experienced airworthiness specialists at the Ministry of Supply, the Board's team, inspired by the late T. R. Thomas, laid the foundations of a set of requirements which have enabled Britain not only to discuss international standards on equal terms with America, but in some respects to take the lead.

How well this has been done is probably expressed best in the words of the leader of the Canadian delegation at the I.C.A.O. Conference in 1947, when in a farewell letter to the leader of the British Delegation he wrote:—

"It is quite impressive to watch the U.K. hewing its way to leadership in the international field of aviation. By preparing a set of national regulations of such technical excellence and in such detail: by introducing P.I.C.A.O. requirements into this set, and stating so, very clearly: by translating these national requirements into French and Spanish, you will, I am confident, soon dominate the field of aviation."

The new civil requirements had a twofold effect on the development of civil aviation in this country. In the first place, during the design of the interim types based on military aircraft, the effect was embarrassing. Modifications were required to aircraft which

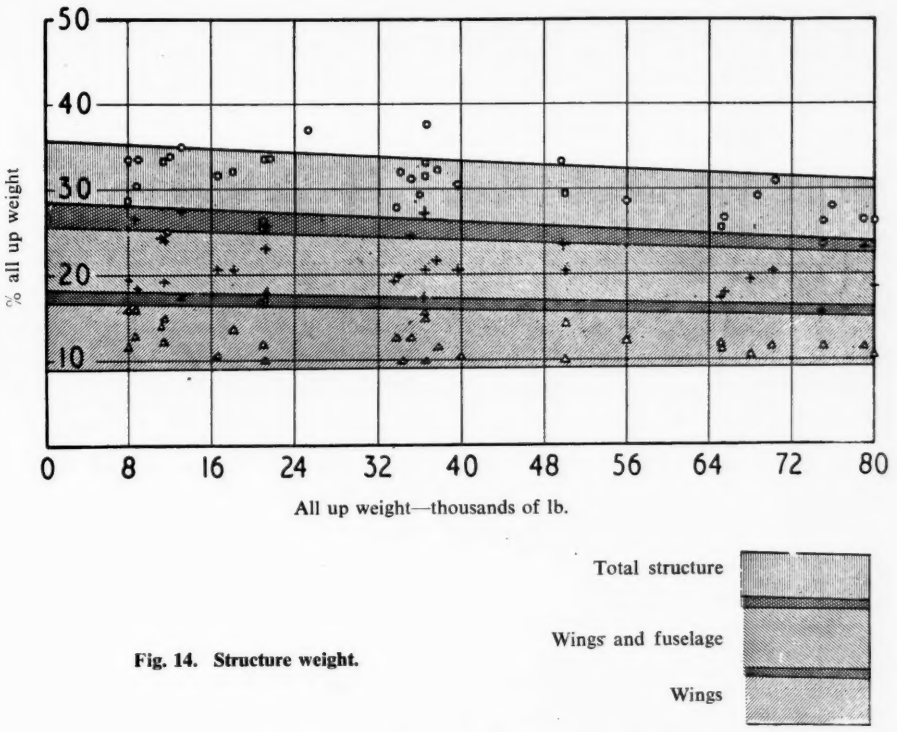


Fig. 14. Structure weight.

had been reasonably satisfactory under military service conditions and a number of small precautions were introduced which were added burdens to the constructors. The fact that most of these requirements were accepted and have stood the test of operational service during the past few years shows the sound logic behind their introduction. The second effect was the general improvement of information available to the Industry, both for structure and performance. It is well known that requirements in general do not play a major part in structural design but they most certainly set standards which must be met and in the author's opinion the availability of clearly written modern requirements at that time was of real value to the Aircraft Industry.

2.4. Stage 4. Military Development

While the major part of the Industry was engaged upon the design of civil types the Air Staff were taking stock of the lessons of war. The enemy's new weapons and interceptor aircraft had created new problems in future policy. Also the jet engine, which was an established success in the fighter types, was only at the beginning of its development; its adaption to civil transport and long-range bombers was still a matter of speculation.

The last development period began when the trend of world politics became unmistakeable and a series of new specifications were issued. This series of aircraft introduced, among others, the many problems associated with high subsonic speeds at high altitudes.

These problems were headed by the new bogy of "aeroelasticity," but the designer was still carrying his three "old men of the sea" from Stage 3 and was becoming conscious that the prop he had welcomed during this period in the form of the new aluminium-zinc-magnesium alloy was at times proving a burden.

At some point in structural papers it is usual to mention, very quietly, the importance of structural efficiency and weight saving, to apologise for so doing and then to pass on quickly.

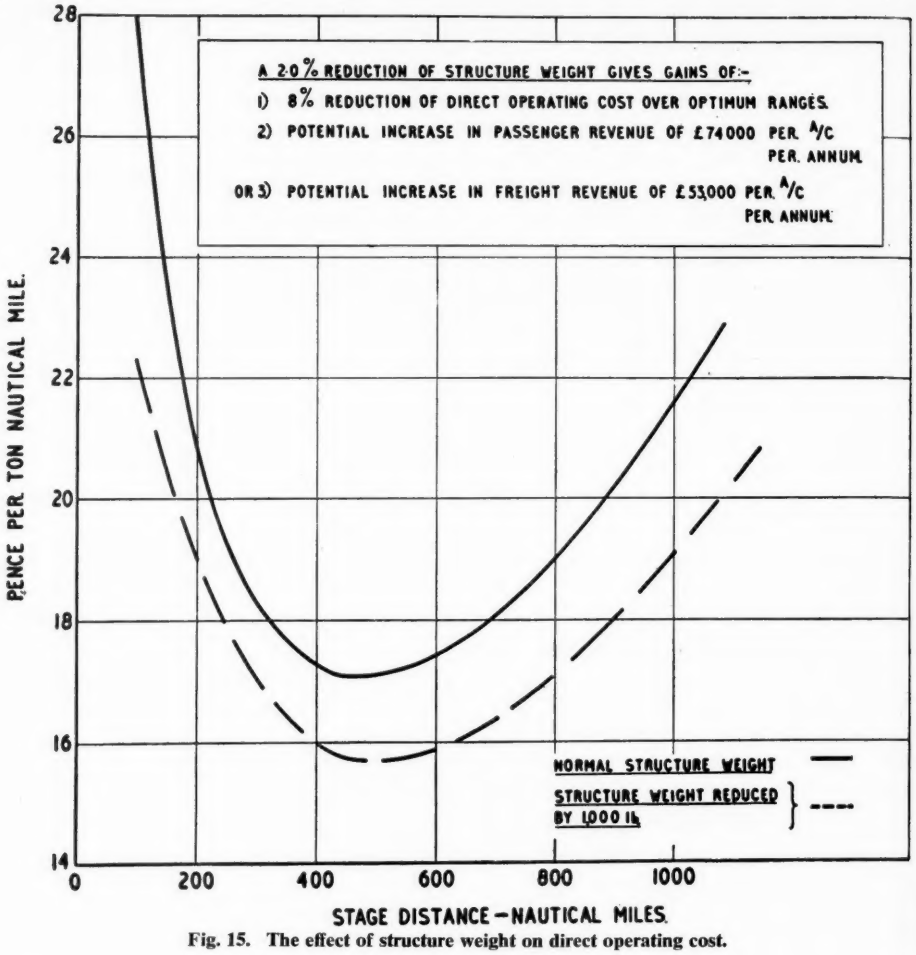


Fig. 15. The effect of structure weight on direct operating cost.

The author thinks this is quite wrong. Structural efficiency is a painful subject—all who delve into the art should suffer by occasionally being reminded of the cost of the lack of this quality.

The need for structural efficiency is greater now than ever before, as the low structure weight necessary to enable the aircraft to reach the ceilings specified has become a more difficult objective because of the changed external shape of the aeroplane. Wing thickness/chord ratios are reduced, engines require to be buried and the undercarriage, which has assumed ponderous proportion, has to disappear. Certainly the structural designer might be excused if he should think occasionally that an Aladdin's lamp is the best solution.

Figure 14, which shows structure weight percentages, has been obtained from data published in current Weight Data on M.O.S. Form 2492, now in general use. It is probable that several of the higher values are incorrect because of the natural development of the aeroplane with the accompanying increase in total weight.

An examination shows, however, that there is a variation of some 10 per cent. in the total structure weights shown, which suggests that there is still much scope for the exercise of

weight control. Variations in the role of different aircraft will account for differences of the order of 5 per cent., leaving, it is suggested, a remaining 5 per cent. in which some reduction is possible.

Values of the component weights of wings and fuselage are shown to indicate some of the difficulties of interpretation as breakdowns differ. The main contributor to the wide margin in total structure weight is the wing structure.

The structure weight percentage is only a first approach in the full assessment of structural efficiency; undoubtedly the effect of wing loading, wing thickness, aspect ratio, the position of engines, the distribution of fuel and other similar basic features must be considered. It will, however, always form a convenient yardstick for a first comparison. As the reduction of structure weights is always a major problem two figures have been prepared to indicate the savings which are possible from a reduction of structure weight.

Figure 15 applies to a civil transport. The assumption was made that a civil aircraft weighing approximately 50,000 lb. has a utilisation rate of 2,000 hours per annum. With the present charges in air transport a saving of 2 per cent. of their weight in the structure will show the following financial gains:—

(*i*) The direct operating cost is reduced by 8 per cent.

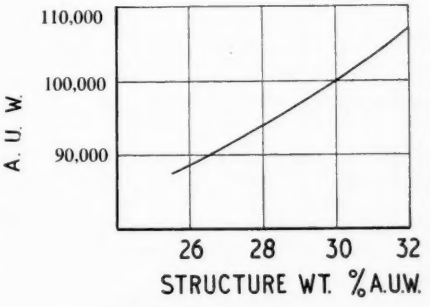


Fig. 16. Military aircraft.
Variation of total weight with structure weight for a given specification.

Structure wt. (% *a.u.w.*)		*A.u.w.*	(% *change*)
34,200	32	107,000	+7
30,000	30	100,000	0
26,300	28	94,000	−6
23,400	26	88,900	−11

(*ii*) The potential earning capacity of the aircraft per annum is increased by £53,000 when the additional weight is carried as freight, or by £74,000 when the additional weight is carried as passengers.

From these figures, assuming a 60 per cent. load factor and a mixed load, the annual earning capacity on a fleet of 100 aircraft would be increased by £3,800,000.

As more than 200 aircraft are at present in service in the British national corporations, it can be seen how important a part structural efficiency can play on the operational side.

Figure 16 shows the effect on a typical military aircraft of varying structure weight. The assumption is made that the design is stabilised before the final change in structure weight can be assessed. On this assumption a saving in structure weight of 2 per cent. would give an overall saving in take-off weight of not less than 6 per cent. If the lower weight could be guaranteed in the original design an even greater overall reduction could be achieved.

On the financial side the saving cannot be assessed as for a civil aircraft but if the annual expenditure on new aircraft is considered, a figure most recently published being approximately £80,000,000, then it can be shown on a weight basis that the same number of aircraft could be put into service with a reduction of approximately £4,800,000. This figure is only a part of the total saving, since a reduction similar to that quoted for the civil aeroplane will appear also in operational expenditure.

These figures are but illustrations of the importance of weight reduction. It is believed that they help to show how the structural engineer, by solving his own major problem, can contribute towards the solution of a much wider national problem.

With these thoughts in mind some of the major problems already mentioned will be discussed.

3. MATERIALS

In all structural problems the material which is used claims its part in the general picture; therefore some consideration has been given to the problems which have arisen in the last two stages regarding the use of metallic materials.

For many years in Great Britain the aluminium-copper alloy had been almost universally adopted for extrusions, forgings

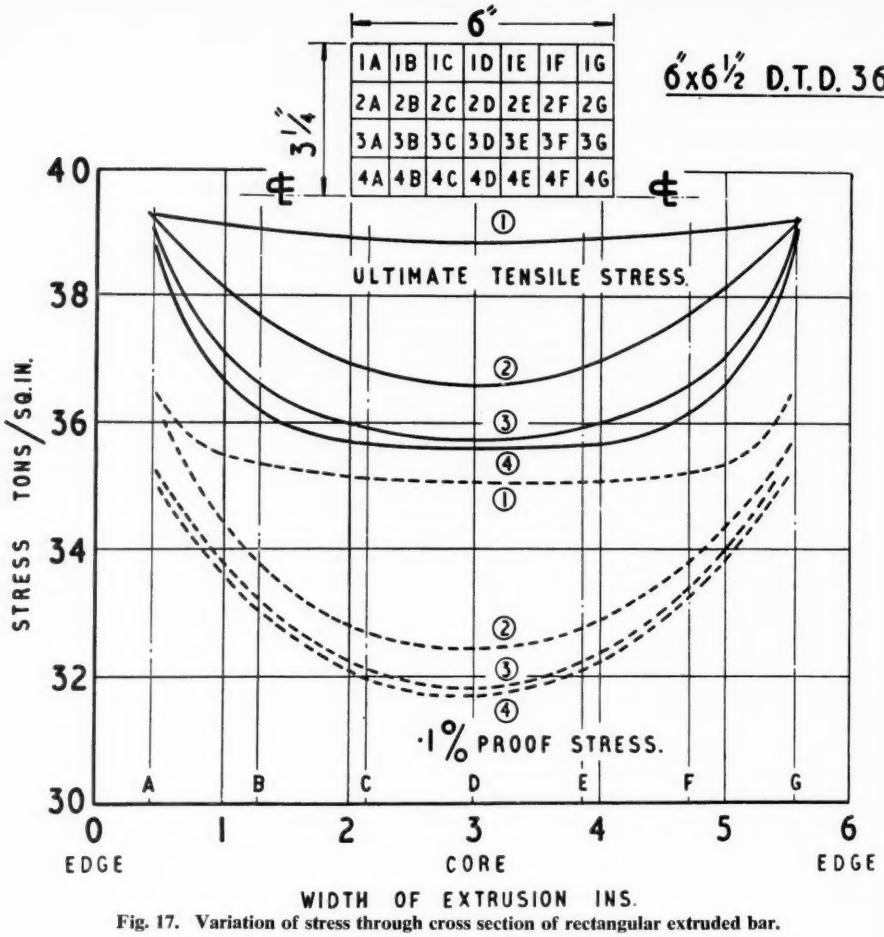


Fig. 17. Variation of stress through cross section of rectangular extruded bar.

and sheet until the higher strengths of the newer aluminium zinc-magnesium alloys, D.T.D. 363 and D.T.D. 683, used as extrusions and forgings, made appreciable weight saving possible. There is always a danger of unexpected troubles when comparatively new and untried materials are adopted in a new design and aircraft manufacturers who are first in the field with a new material can invite a wasteful amount of trouble in correcting unexpected defects. Whenever the gain appears sufficient to justify this risk the new specification must be adopted. Its introduction in stages appears ideal but this is not usually effective as the small part will seldom give any real indication of the troubles which can occur with the much larger components.

With the aluminium zinc-magnesium alloy its general adoption started in quantity with the new civil aeroplanes and in the last period under discussion its use has become the rule rather than the exception.

The turbo-jet engine, which is buried in wings of the larger types, has proved in some ways an obstacle to the steady advance of the idealised shell construction. Access doors must be provided and large holes are necessary in the leading and trailing edges of the main structural box; therefore heavy spar booms have been retained in the present generation of large military types at a time

when, but for the problem of the buried engine, the size of spar booms would have been appreciably reduced and a greater use of the orthodox stringer-skin construction would have been possible.

The extreme size of extrusions has been found desirable to meet the requirements of the joint. Experience has shown that the use of steel couplings to join two booms together is extremely wasteful in that the joint is duplicated while the strains in corresponding parts of the joint cannot easily be matched. For these reasons, among others, the design of joints by machining mating parts from the solid appears to have many advantages which outweigh the complications incurred by the size of the extrusion.

In sections where the ruling thickness is of the order of 6 in. two difficulties have arisen,

(*a*) distortion after machining,

(*b*) variation of strength across the section.

The distortion problem with this alloy has become of general importance as it is not confined to extrusions. With extrusions a controlled stretch in the solution-treated condition immediately after quenching has reduced the trouble, but with highly tapered spar booms it is advisable to machine in the solution-treated condition in order that any correction which is necessary may be made before precipitation. This is an unsatisfactory aspect of the new alloy.

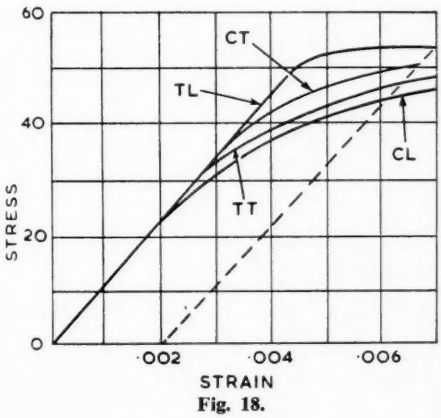


Fig. 18.

T_L = Tension (longitudinal)
T_T = Tension (transverse)
C_L = Compression (longitudinal)
C_T = Compression (transverse)

The second effect, which gives low core properties, is one which must be known before design values for the material are agreed. Fig. 17 shows the variation of stress which occurs across the boom and indicates some 6 to 7 per cent. reduction, which must be accepted.

In the forgings or large forged billets of D.T.D. 683, teething troubles have not been absent. The distortion occurring when the fully heat-treated forging is machined has made this approach impractical in the larger sizes. It has been found necessary to rough-machine in the annealed condition before fully heat-treating and to allow a rest period of some 30 hours before machining to size. The lack of stability shown in both extrusions and forgings is an unpleasant feature in production.

3.1. The Effect of Grain Direction

A variation of strength between the longitudinal and transverse grain directions has been known to exist for a considerable time. Its effects, until some two years ago, were considered small and stress reductions of the order of 10 per cent. only were made to tensile stresses across the grain. The unexpected failure on test of large fittings loaded in a transverse direction led to a series of tests on tensile specimens cut from large sections of extruded bar and forged billets.

The results summarised in Table I indicate the allowable stresses which can be used with and without stress concentrations.

An interesting comparison with these results is the effect of transverse grain on the strength of sheet as shown in Fig. 18[2], where the compressive strength in the transverse direction is higher than that in the longitudinal direction.

Some of these effects are met in tests on fittings made from forgings or bar where unexpected failures in the transverse direction occur at stresses which can only be explained by high stress concentrations. These failures have unfortunately been all too frequent, resulting in the official forging test requirement.

3.2. Steel Forgings

Large steel forgings are used in many undercarriages or their attachments; the smaller bulk still makes steel more attractive than light alloy in installations with space

limitations. Unfortunately their production is limited, and the number of cracks which appear in large components has become a major defect. Usually a fitting is fully machined before the cracks are discovered and the fitting scrapped. Thus not only is expense much increased but serious delays in prototype aircraft can be caused.

It is certainly recommended that improvement of forging technique is a matter of high priority.

3.3. Low Temperature Considerations

With the high-speed, high-altitude aircraft designed to fly for long distances, little is known of the steady temperatures which will exist in cruising conditions. At heights of 50,000 ft. ambient target temperatures of $-90°C.$ are quoted while $-75°C.$ is considered to be essential as a design parameter. Thus, until aircraft have flown under these conditions and temperatures have been measured, some consideration must be given to the effect of low temperatures on materials used in stratospheric aircraft.

In general the reduction of temperature increases the mechanical properties of most metallic materials, but with many steels the serious increase in notch sensitivity gives rise to grave doubts of their suitability at low temperatures.

The following are of importance to the structural designer:—

Carbon steels (B.S. S.1, S.93, S.84, etc.)
Ferritic steels of low nickel content (some B.S. S.95)
Ferritic chromium steels (B.S. S.61, S.62, S.80).

It is considered that these steels should *not* be used where they are exposed to the full effects of low ambient temperatures, a consideration which is of some consequence, particularly as it practically eliminates the use of mild steel and throws doubt on many stainless steels.

The complete abandonment of mild steel for aircraft for all structural purposes can be borne without serious penalty. Already the general standardisation of high tensile steel nuts and bolts has removed a large proportion of this class of material. Therefore, at low temperatures, the danger of the type of failure which in the past has been associated with high tensile steels such as B.S. S.28, has been rendered unlikely. The elimination of the remaining mild steel, now used in small welded component structures, could also be done without any real penalty.

With low temperatures the effect of notch fatigue endurance has not been seriously explored. Some work of a reassuring character has been completed, but when consideration of work hardening and crack

TABLE I

	Material	Manufacturer	Ultimate tensile stress: ton/in.² Longitudinal "L"	Transverse "T"	Cross-grain "C"	Transverse longitudinal	Cross-grain longitudinal	Percentage elongation Longitudinal "L"	Transverse "T"	Cross-grain "C"
Plain Specimen	D.T.D. 683	A	36.19	31.82	28.56	0.878	0.788	6.0	2.0	1.0
	D.T.D. 683	B	36.82	29.28	29.35	0.793	0.796	12.5	2.0	3.0
	D.T.D. 364	A	28.14	23.91	24.10	0.850	0.856	6.5	2.0	3.0
	D.T.D. 364	B	29.39	22.68	22.13	0.772	0.753	8.0	2.0	2.0
	D.T.D. 423B	A	24.83	17.96	19.00	0.725	0.763	12.0	—	3.5
	L. 39	A	30.32	20.04	22.69	0.658	0.750	17.5	4.0	4.5
Hole Specimen	D.T.D. 683	A	35.69	24.30	25.67	0.680	0.720	2.5	1.0	1.0
	D.T.D. 683	B	27.64	19.66	20.44	0.710	0.740	2.5	2.0	2.0
	D.T.D. 364	A	28.01	23.14	21.44	0.825	0.765	4.0	1.5	2.0
	D.T.D. 364	B	29.91	17.07	21.10	0.570	0.706	7.0	4.0	4.5
	D.T.D. 423B	A	24.88	19.11	19.58	0.768	0.786	8.0	2.5	2.0
	L. 39	A	29.91	17.07	21.10	0.572	0.706	7.0	4.0	4.5
Notch Specimen	D.T.D. 683	A	33.96	19.29	19.83	0.568	0.584	1.5	1.0	*nil*
	D.T.D. 683	B	31.84	23.70	19.34	0.744	0.607	2.0	1.5	1.5
	D.T.D. 364	A	24.28	14.77	18.16	0.608	0.747	2.5	1.5	2.0
	D.T.D. 364	B	26.56	16.59	19.28	0.625	0.726	2.0	1.5	1.5
	D.T.D. 423B	A	23.59	15.98	19.15	0.677	0.812	3.0	2.0	2.0
	L. 39	A	27.69	19.24	20.31	0.695	0.733	3.5	3.0	3.0

TABLE II

SHEET MATERIAL

Material	*Specification*	*Ultimate tensile stress: ton/in²*	*Density: lb/in³*	*E ton/in²*	*0.1% Proof stress: ton/in²*	*Strength/ weight ratio: comparison with D.T.D. 124*
Aluminium alloy	D.T.D. 646	28.0	0.1015	4,470	23.0	1.75
Magnesium	D.T.D. 626	17.0	0.0647	2,900	11.0	1.67
Stainless steel	D.T.D. 166	52.0	0.286	12,700	40.0	1.16
High tensile steel	D.T.D. 124	45.0	0.286	12,700	40.0	1.0
Mild Steel	S. 3	28.0	0.286	12,700	16.0	0.623
Unalloyed titanium:						
Annealed		35.7	0.162	6,700	32.2	1.40
Half hard		51.5	0.162	6,700	46.7	2.02
Full hard		55.8	0.162	6,700	49.0	2.19

EXTRUSIONS

Alloy	*Approximate percentage composition or specification*	*Condition*	*Density: lb/in³*	*Ultimate tensile strength: ton/in²*	*E ton/in²*	*Elongation in one inch: per cent*	*Strength/ weight ratio: comparison with S. 99*
Aluminium alloy	D.T.D. 363	Fully heat treated	0.1015	38.0	4,470	5.0	1.34
Aluminium alloy	D.T.D. 364	Fully heat treated	0.1015	32.0	4,470	8.0	1.13
Magnesium alloy	8.0 Al. 8.0 Cd. 2.0 Ag. remainder Mg.	Fully heat treated	0.0647	30.6	2,770	2.4	1.69
High tensile steel	S. 96	Fully heat treated	0.286	55.0	12,700	18.0	0.69
High tensile steel	S. 99	Fully heat treated	0.286	80.0	12,900	14.0	1.0
Tungsten-carbon-titanium	4.7 W. remainder Ti+C+N	As forged	0.162	88.4		2.0	1.95
Chromium-tungsten-titanium	10.3 Cr. 0.79 W. 0.27 C.	Heat treated and quenched	0.162	73.4	Varies from 8,040 to 6,250	2.0	1.62
Chromium-nickel-titanium	5.0 Cr. 2.0 Ni. 0.2 N.	Hot rolled	0.162	86.9		3.0	1.92

propagation are given their full importance it will be realised that there is much cause for anxiety. Thus the need for further evidence becomes one of some urgency.

3.4. TITANIUM

Titanium is the most important at present of the new materials; its success will depend on how soon it becomes available. In the jet engine field it seems to have pride of place because of its ability to withstand high temperatures, while its resistance to corrosion, its weldability and low specific weight make its availability extremely attractive.

Its strength and weight relative to other materials are shown in Table II.

While small quantities for experimental work are required as a matter of extreme urgency in this country, the possibility of its general adoption for the structure of supersonic aircraft cannot be too carefully considered. It is recommended by the author that, in parallel with the design of these

aircraft, the production of suitable alloys of titanium and the investigation of methods of fabrication should proceed on very high priority.

4. PRESSURISATION

The structural problems created by the introduction of the pressure cabin are chiefly associated with departures from the cylindrical or spherical shape. Openings for pilot's hood, doors and windows, unorthodox bulkheads and floors under pressure loads, all produce their own peculiar difficulty. Often the solution is provided by "brute force" checked by the test, but frequently a more mathematical approach can be made.

An example of this approach is provided by the elliptical window where it has been shown, with a cabin of circular section, that by designing an opening to certain elliptical dimensions, the edge member is relieved of bending loads, which would normally occur, due to the pressure effects on the surrounding skin. This theory was developed mathematically by B. Stephenson, of Vickers-Armstrongs, and confirmed by pressure tests on a specimen suitably strain-gauged. The shape of the Viscount doors and windows are examples of the introduction of this structural feature. Because of the curvature of the cabin shell the boundary member to an opening requires to be a torsion box. This is achieved in the window (Fig. 19) by the introduction of a top hat section as the boundary member. With a main door opening the design of the boundary member becomes more complicated because, in addition to the larger "out of the plane" loads, stiffness is of paramount importance for satisfactory sealing and easy operation of bolting mechanism. Floating hinges have to be provided to allow even seating of the door seals and to prevent the imposition of heavy concentrated loads.

Another example of the mathematical approach is the recent analytical treatment of the front end bulkhead of a military aircraft. This bulkhead was required to be concave in place of the usual convex hemispherical shape. The structure of the bulkhead consisted essentially of a central hub from which radiated built-up arms to the outer boundary ring, with panel segments stretched between the arms. In an attempt to keep weight to a minimum it was stipulated that the arms should be loaded in compression only and that the plating should be subject to tension only. The mathematician provided a solution for the design of the compression members, while a "free blown" technique provided an answer to the segment shapes. The technique involves the adoption of surfaces with the shape which would be taken up by an elastic diaphragm such as a thin rubber membrane when subjected to pressure. For the case in mind a sheet of heated Perspex was "free blown" over representative edge members. Moulds were then made from the Perspex pattern and the duralumin panel finally pressed. Care had to be taken to allow for the contraction of the Perspex on cooling. An additional instance where this method has been successfully used is for the cockpit flooring, under which an unpressurised bay had to be provided The result was a series of catenary-like panels resting on the floor beams. This practice of avoiding flat surfaces where cylindrical or spherical shapes cannot be used shows great promise, both in the avoidance of large deflections and in weight saving.

To meet strength requirements pressure cabins have to be designed to carry twice the maximum operating pressure, in other words a factor of two is required. Calculations will not ensure that this can be achieved, therefore the *ad hoc* test must be made. The hydraulic tank method of testing has given designers facilities for checking pressure cabin strengths perhaps safer than most of those provided by the mechanical test laboratory or the full-scale structural test gear. The introduction of this method constituted a big advance by comparison with the earlier method of testing with air pressure.

With this method the test specimen can be kept under observation during the application of pressure, and the test can be stopped at any point if excessive deflections or bad leaks require investigation. After a failure in the tank the damage to the specimen is usually local, thus enabling repairs and further tests to be made.

The effect of repeated loading is being given attention. Since most pressure cabins have to carry main structural loads as well as pressure loads the possibility of fatigue failure is a definite anxiety. The comparatively short time during which pressure cabins have been in operation does not enable an opinion to be expressed on the extent to which this problem has been solved.

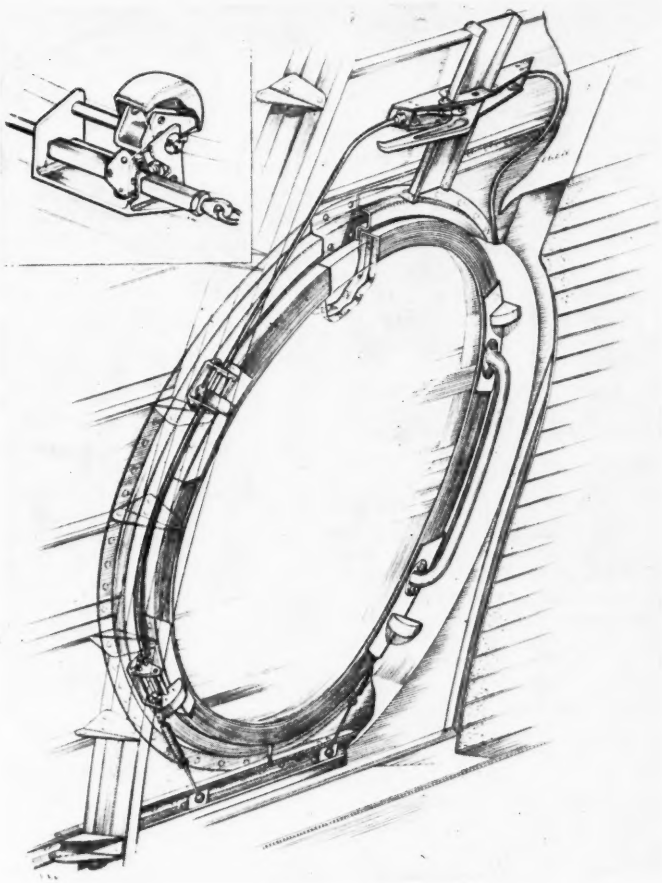


Fig. 19.

Many components have been tested under repeated loading but very limited work has been done on complete cabins.

In new aircraft the repeated loading test on the cabin specimen is featured in the structural test programme. Evidence from these tests should enable any local defects to be corrected and should prove valuable in assessing the strength reserves required.

The problems associated with pressure unfortunately are not confined to the pressure cabin; the maintenance of the fuel supply at altitude is of the same order of importance as some of the structural problems. With liquid fuels now available in quantity the maintenance of the fuel supply also becomes dependent upon pressurisation.

The more volatile fuels with high Reid vapour pressures (R.V.P.) are susceptible to reductions of surface pressure and will boil readily; large quantities of fuel will be lost unless suitable preventive measures are taken. Since temperature and pressure are both involved in this problem a choice between refrigeration or pressurisation must be made and since the refrigeration is considered to be operationally impracticable, pressurisation of fuels becomes essential.

Figure 20 shows the relationship between altitude and the pressure difference required to suppress boiling for various initial temperatures of fuel. It covers fuels having Reid vapour pressures of approximately 7 and 3. It will be seen that the lower vapour

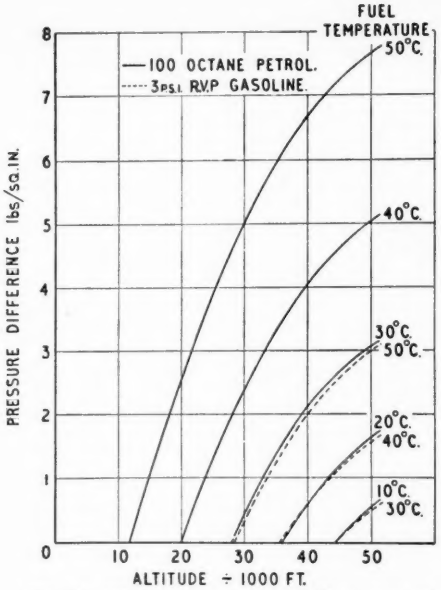


Fig. 20. Tank pressure above ambient required to suppress boiling.

pressure fuel at 50,000 ft. requires a differential pressure of 3 lb./in.^{2} to suppress boiling. If negative pressures are now considered on the upper surface of a wing in the high-speed case shown on Fig. 21, it will be seen that suction of about 3 lb./in.^{2} is also present; thus the structure of an integral tank or the supporting structure of a thin bag tank must withstand pressures of the order of 6 lb./in.^{2}. Reference again to Fig. 21 will show also that the position of the vent which finally decides the maximum pressure is important.

With this brief example in mind it can be seen how important the design of fuel tanks has become. When it is considered how far structural engineers will go to ensure minimum stresses in a pressure cabin, hood or blister, it is apparent that the pressurised fuel tank is not usually given its full share of attention.

If the values with the higher Reid vapour pressures were, in fact, demanded without loss of fuel it can be seen how impractical the whole problem becomes.

5. TRANSPARENCIES

"The future development of British aircraft depends upon transparent materials as much as it does, for example, upon supersonic aerodynamics or the jet engine."

A statement of this nature was made some two years ago and supported by arguments which showed how necessary the improvement of optical radio and radar transparencies had become.

Structural workers will not deny the truth or the seriousness of these remarks and, although the past two years have seen the fruits of some research, all will agree that the need for a transparency which is nearer a structural material grows greater as speeds increase.

The optical transparencies were the first to receive the attention of the designer, as work on the later civil transport aircraft demonstrated the problems of the windscreen and the window; thus the relative merits of glass and Perspex were subjected to the closest investigation. At this time also, the dangers of explosive decompression were accentuated and the danger to passengers in pressurised aircraft due to window failure received much publicity. As the whole problem is still far from being resolved satisfactorily it is thought that some detailed discussion on both Perspex and glass is warranted. The radio and radar transparencies each present their own particular problems of an even more specialised nature; therefore, although the needs for improvement in this field are equally great, further detail on these points is not given here.

5.1. PERSPEX

When the problem first became prominent, knowledge on the basic mechanical properties and their variation with temperature was

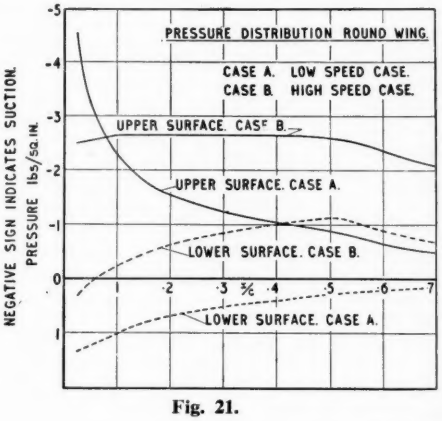


Fig. 21.

limited. R.A.E. Report Struct. 35/Chem 453 gave the following information about plasticised Perspex for a range of temperatures from $-60°C.$ to $+60°C.$

Ultimate tensile strength	12,200 — 2,000 lb./in.^{2}
Youngs modulus	500,000 — 100 lb./in.^{2}

with a coefficient of expansion 8 times that of steel or $3\frac{1}{2}$ times that of light alloy. The importance of high differential expansion has been shown in the failures experienced on fighter canopies. Inquests on these failures had shown that in nearly all cases a failure could be attributed to the combined effect of differential expansion and notch sensitivity.

Notch sensitivity has been shown in test programmes for cabin windows. The dry air sandwich which has been used without trouble on unpressurised aircraft, when tested under pressure shattered at low stresses and showed a large amount of scatter in the results. Also, any form of hole or notch proved to be quite unacceptable because not only did the presence of stress raisers initiate failure, but it also made the estimation of strength impossible.

In this way evidence was obtained to show that Perspex windows must be mounted flexibly and must be free from any form of notch or hole.

Repeated loading tests on the windows gave rise to many doubts on the reduction of stress which was likely with this form of loading. Therefore a rather extensive programme was considered necessary to check the mechanical properties of the material, with particular reference to repeated loading.

From a research programme undertaken by Vickers-Armstrongs at Weybridge, some results are now available. These emphasise the low strength at high temperatures and bring out very forcibly the effect of creep. They show also that prolonged loading at normal temperatures has no serious effect, even though severe crazing is apparent. The most important feature, however, is the effect of repeated loading. Tests at 20°C. and 55°C. were conducted with loading periods of 8 hours, followed by rest periods of 16 hours, for 400 cycles, representing a total loading time of some 3,200 hours. At 20°C. no crazing could be detected on specimens loaded below 1,800 lb./in.^{2}, while at 55°C. no crazing occurred below 1,600 lb./in.^{2}. The stresses corresponding to these two temperatures at 20°C. and 55°C. were 3,400 lb./in.^{2} and 3,000 lb./in.^{2}. These results indicate that, provided no stress concentrations are present, the average Perspex window designed to the low value of 1,000 lb./in.^{2} has in fact an ample margin. The effect of solvent crazing remains a much more difficult problem, because at room temperatures crazing of this type will be present at stresses above 500 lb./in.^{2}. The uncertain nature of the material has led to official requirements being introduced on the lines of those issued for castings.

The factor which must be achieved is related to the number of test specimens; at the present time the factors vary between 8 and 6.8, while the number of Class I specimens required varies from one to 6. This level of strength is now backed by a considerable amount of laboratory testing which indicates that the factors chosen are reasonable.

When the cost of canopies is considered, the financial burden imposed by the official requirements becomes considerable. It has been found that an electrical strain-gauging technique can be adopted and will permit the determination of strains under maximum load conditions. It is believed that if sufficient exploratory work using strain gauges is undertaken, then the need for large numbers of tests will be obviated and a single confirmatory test on one specimen to prove the ultimate factor will be all that is necessary. This technique is considered of sufficient importance to warrant its further development and probably its general adoption—*provided that there are no built-in stress concentrations.*

5.2. Glass

As an engineering material glass suffers from disadvantages similar to those of Perspex and in addition has peculiar difficulties of its own. It will not withstand tension to any degree and as the Youngs modulus is high (8×10^6 lb./in.^{2}) the high pressures of the more recent pressure cabins call for particular care in mounting, in order that cracks may be avoided. Although it is twice the weight of Perspex it will stand abrasion and is not affected by solvents. In the three types in common use, annealed, semi-toughened and toughened, ultimate strengths of 6,500, 18,000 and 23,000 lb./in.^{2} are quoted, the last two values being achieved by the introduction of compressive stresses during the cooling after heat

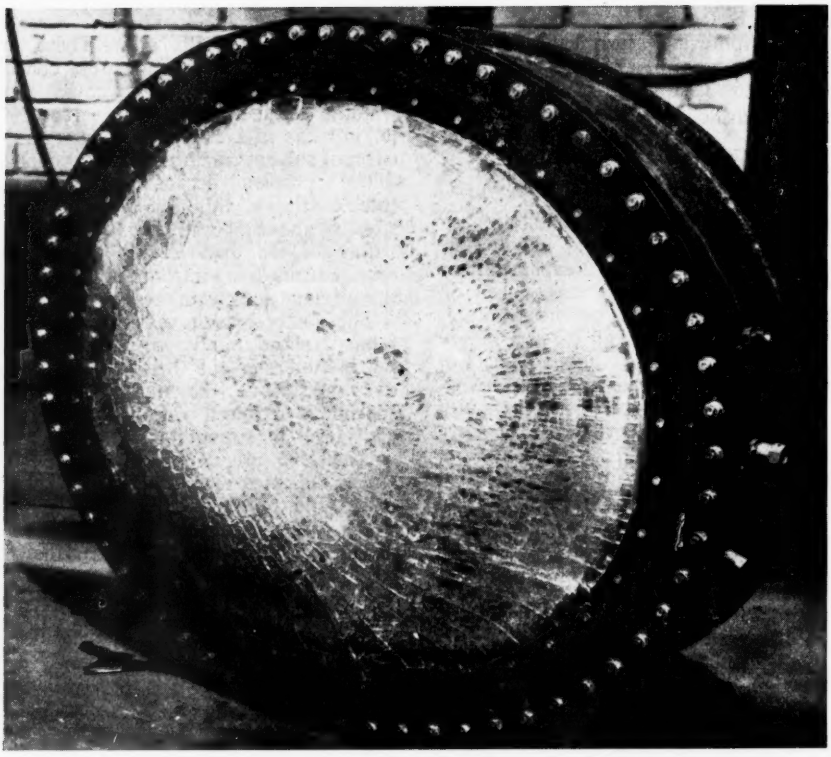


Fig. 22.

treatment. These surface stresses, engendering high internal stresses, cause toughened glass to become opaque on failure while the locked-in stresses give the failure an explosive character. For this latter reason lamination by means of a vinyl interlayer has become mandatory in most applications.

The use of the extended interlayer has made possible the method of mounting, now universally adopted, which permits a degree of flexibility, thus isolating the windscreen from the frame. That portion of the vinyl interlayer projecting beyond the edge of the glass carries a metal strip, through which attachment bolts are fixed, allowing the whole panel to be carried on the interlayer in a flexible manner. Should the glass fail the interlayer can thus act as a membrane and retain the pressure (*see* Fig. 22). Even with these precautions glass windscreens are constantly being changed because of cracked panels or de-lamination at the edges (a common fault). Official requirements admit the uncertain nature of the material by specifying factors of 10 for components made from annealed glass and 6 for components made from semi-tempered or fully-tempered material. This position, as revealed by the high factors made mandatory, shows how unsatisfactory both Perspex and glass must be considered when viewed solely as aircraft structural materials. That satisfactory panels for both windscreens and windows have been produced must be ascribed entirely to the detailed care which is taken in the design and testing of the panels. Bearing in mind the fact that glass has a specific gravity of 2.38, very nearly equal to that of light alloy, and that the specific gravity of Perspex is 1.19, these values, viewed in the light of the very low working stresses which can be permitted, indicate the weight penalty involved by the use of present types of glass and acrylic materials.

It is, however, considerations of the future use of the transparencies in supersonic aircraft which are particularly alarming. If by outlining briefly the present position the author has directed more attention to the problem, it is possible that this section may be justified.

6. THE FATIGUE PROBLEM

The importance of fatigue to the structural engineer dealing with civil aircraft increases almost alarmingly as experience is gained, both from the laboratory test and from the long-term evidence provided by many hours of operational flying. The evidence from the laboratory tests is mostly of a pessimistic nature in that the fatigue test machine will produce failures at stress levels much lower than are known to be accepted in common practice. The evidence from operational flying is difficult to assess. It appears to give a much more reassuring picture in that few failures have occurred, the majority of these being of a secondary nature. Isolated instances of catastrophic failure, however, prove to the designer that the problem becomes more serious as present-day aircraft continue to build up flying hours, particularly when full account is taken of the gradually increasing utilisation rates made possible by the increasing efficiency of the operator.

Utilisation rates of between 2,000 hours and 4,000 hours per annum are being realised at the present time by aircraft for which a useful life of fifteen to twenty years has been predicted. As aircraft with flying lives exceeding 40,000 hours are still in service the full magnitude of this problem can be fully realised.

Few theories on the basic nature of fatigue from the point of view of metal physics have been advanced and that given by P. L. Teed[3] is a notable contribution. In this lecture the propagation of the fatigue crack within the crystal boundaries of the material is explained. Notwithstanding theoretical advance and the experience of many years of laboratory testing, present knowledge does not enable the prediction of the safe life of the aircraft components to be made with any degree of certainty. For this reason it is necessary to resort to some arbitrary standard whereby the results of fatigue tests can at least be judged on the basis of comparison and from which some assurance of a satisfactory operational life can be gained. Work carried out at the Royal Aircraft Establishment and elsewhere has led to the present suggestion that a suitable criterion for such a purpose is the application of axial tensile loads of $1\,g \pm 0.075 \times$ the Ultimate Design Load for 2×10^6 cycles. Recent work has shown that this criterion is too severe as it is possible to achieve such a standard only by great care in the initial design and a very high standard of workmanship in the finished part. Research programmes are in hand in which a number of typical joints are being tested under the foregoing loading conditions and it is hoped that before long it will be possible to state the number of cycles which will ensure a satisfactory life. It is feared that the programmes which must be completed before any degree of certainty can be achieved will not only be long and tedious, but must eliminate some of the present uncertainties regarding fatigue testing machines and size effect.

At present the variation of fatigue test results obtained on different machines is in some cases much greater than the scatter in the test results from the same machines. Also, it is known that the small part, or scale model, tends to give better results than the larger, full-size specimen. Both these facts will need the closest investigation before the full problem of fatigue is resolved.

Considering now the criterion of $1\,g \pm 0.075$ Ultimate Design Load just mentioned, an indication of the standard demanded, if 2×10^6 cycles are to be achieved, can be obtained from a study of the test results from fatigue tests on a simple lug-type joint which can be considered typical of current practice. These are shown in Fig. 23.

The specimen is made from D.T.D. 364 material, the lug being 2.25 in. wide $\times$ 0.75 in. thick with a 0.75 in. diameter pin fitted with a light driving fit. The full curve is a fair curve showing the results of 22 tests at a mean stress of $2\frac{1}{2}$ ton/in.^{2}; approximately half of the tests were done in the Losenhausen machine at the College of Aeronautics and half in a Schenck machine at the National Physical Laboratory. It will be seen that a life of 2×10^6 cycles is obtained only with alternating stresses less than 1.0 ton/in.^{2}. On this basis, if the criterion is to be met, the Ultimate Design Stress should not exceed $1/0.075 = 13$ ton/in.^{2}, which is less than half the ultimate strength of the material. If the number of cycles required were reduced to a half, *i.e.* to 1×10^6 cycles, the ultimate allowable stress would then be $1.3/0.075 = 17.3$

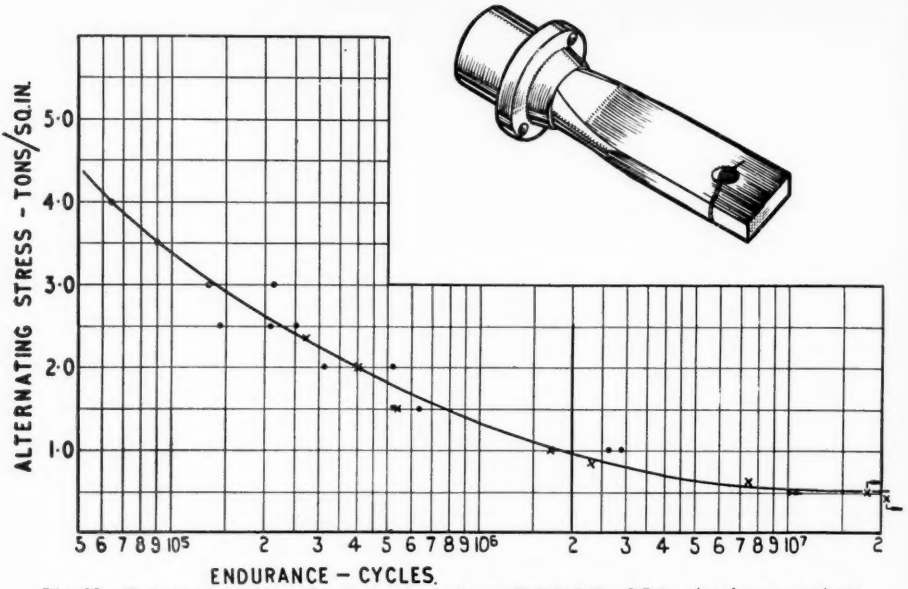


Fig. 23. Fatigue test results. Loaded hole specimens D.T.D.364. 2.5 tons/sq. in. mean stress.
• College of Aeronautics M/C (Losenhausen). × N.P.L. M/C (Schenck).

tons/in.². As these ultimate permissible stresses from this test are based on a mean stress of 2.5 ton/in.² it is clear that the lug test provides another indication of the pessimistic results obtained from the laboratory test.

It should be noted that the specimens for this series of tests were finished to good production aircraft standards. Better finish, such as high-grade polishing of the lugs, would undoubtedly have given better values, a point which emphasises the importance of workmanship in dealing with improvement in fatigue strength. Advances in fatigue design will have to be matched by advances in workshop practice.

It has long been established that the fit of pins in a joint is very important from the fatigue aspect, taper pins, for example, showing marked improvements over parallel pins fitted to normal standards. Recent work at the R.A.E. by W. A. P. Fisher on the "loaded hole" specimen mentioned previously, has shown that an interference of 0.0025 to 0.003 in. in the fit on pins of $\frac{3}{4}$ in. diameter can increase the endurance of a test joint as much as four times. This is considered to be an important contribution to the work on fatigue and consideration must be given to the question of obtaining interference fits of this order in the major structural break joints, while at the same time having a joint which can be broken down quickly when required.

The magnitude of the endurance changes which can be effected by comparatively minor changes of design gives rise both to hopes for possible improvement in detail design and to doubts regarding the effect of the normal engineering tolerances of workshop practice. Each of these effects needs the closest examination. It has also been established that fatigue life at the criterion loads is improved by the application of a large number of small alternating loads, often referred to as "understressing," and that, on certain types of joint, "overstressing," *i.e.* a single application of a load of the order of 50 to 70 per cent. of the Ultimate Design Load, has an extremely beneficial effect on the endurance of the joint on subsequent fatigue test. It is interesting to note here that in carrying out the routine pressure test on a pressure cabin it may be that its fatigue characteristics are, in fact, being improved, while unquestionably a higher load of the order of 2 *g* in flight tests of a new aeroplane will have similar beneficial effects.

The major problem in correlating the results of laboratory fatigue tests and the flying life of an aeroplane arises from the fact that, at best, the laboratory test can be done only under a limited range of loading conditions and most conveniently under one condition, such as 1 $g \pm 0.075$ Ultimate Design Load, whereas the actual loading conditions on the aircraft are completely random about the 1 g load which, itself, varies throughout the duration of any flight. The so-called Cumulative Damage Law offers the only reasonable means of correlation at present and two points which contradict it have already been mentioned. Its rigid application therefore must be regarded with considerable suspicion.

In order to use the Cumulative Damage Law or any other which may be found to replace it there is an urgent need for extensive flight measurement of the loads experienced by various types of aircraft. In this country two instruments have been developed which offer prospects of rapid improvements in this direction. These are

(*i*) the counting accelerometer,

(*ii*) the counting strain gauge.

A limited amount of evidence has been obtained from research with these instruments on Lancaster and Viking aircraft at low altitude and the British European Airways Gust Research team has produced evidence from high-altitude flights by other means. It is suggested that the installation of the statistical equipment on operational aircraft, particularly the modern types, is an urgent necessity. The information to be obtained thereby would help to tie together many of the loose ends in knowledge of the fatigue problem.

7. AEROELASTICITY

Of the problems which have greatly increased in magnitude and complexity with the introduction of sweepback combined with the thin wing sections of high subsonic speeds, none has become more important than that of aeroelasticity. The true importance in the purely structural field of this problem with the slower aircraft has been obscured for many years by the general use of thick wing sections clad with metal skin of nominal gauge.

The arbitrary criteria of official requirements had presented no particular difficulty, nor had they imposed any serious weight penalty on wing structures.

With the thinner tail surfaces then in use more attention was required to ensure that the skin gauges necessary for bending and torsional strength also provided the required stiffness. The additional amount of calculation required to check that the stiffness of the various components satisfied all requirements, was not excessive and could be undertaken as a routine task in the stress office. With the advent of high-speed aircraft this comfortable situation disappeared, the simple criteria became millstones almost overnight and "Aeroelastic Groups" or Flutter Sections became necessary, for which mathematicians, particularly graduates with good degrees, became ideal recruits. A more realistic approach became essential.

Professor Collar, in two papers to the Royal Aeronautical Society[4, 5], did much to emphasise the need for this and pointed out that these criteria were never intended to be anything other than "yardsticks by which, in the early stages of design, the elastic properties required for an aircraft can be estimated without serious error and by which the final design can be compared with other aircraft." The more realistic approach involves the full calculations, a procedure which will be possible only if firms have adequate staff for the purpose.

The phenomena to be investigated are of two types, which have been described as the oscillatory and non-oscillatory types. Flutter is the most important of the oscillatory types but in recent years "transient stressing" has entered the field, although some may object to its being classed as an aeroelastic problem. It does, however, fit the aerodynamic-elastic-inertia triangle (*AEI*) used by Professor Collar to illustrate the wide diversity of subjects which should be grouped under the heading "Aeroelasticity." Reversal of control and divergence belong to the non-oscillatory type of aeroelastic phenomena, as does loss of longitudinal stability arising out of the effects of structural deflection.

It is seen that the field to be covered is wide and in the past the practice was to take advantage of recommended stiffness criteria to clear the flutter, control reversal and divergence problems. The influence of aeroelasticity on stability has only become critical with the advent of the swept-back wing. Transient stressing has grown from the use of low normal acceleration factors for aircraft having high aspect ratio wings and comparatively high gust acceleration factors and from

the failures of engine mounting on certain aircraft under landing and taxying conditions. This aspect of the problem gives rise to the possibility of objection to its classification as an aeroelastic problem as in this case there are no aerodynamic forces involved. These two cases of transient stressing come into the points *G* and *Z* respectively of Professor Collar's triangle.

If the full investigations are to be made with the maximum economy from the weight point of view, firms engaged in the design of high-speed aircraft must develop the specialist groups to deal with aeroelastic considerations, such groups working hand in glove with the stress offices in order that there may be a proper correlation between strength and stiffness. The words are those of Professor Collar and the weight of them has increased since they were written, because stiffness rather than strength tends to dictate the structural design of the advanced aeroplane.

It is now proposed to give some consideration to the individual components as affected by the various aeroelastic phenomena.

7.1. WINGS

7.1.1. *Flutter*

In recent years flutter has been the major aeroelastic problem in wing design. The recommended wing stiffness criterion still appearing in A.P.970 and the B.C.A.R. handbook is not suitable for current use as it does not take into account certain favourable parameters, the most important of which are the influence of the position of the inertia axis and of wing taper. In January, 1946, a report[6] was published, giving the results of an R.A.E. investigation and recommending a modification of the criterion to take account of these factors. It is surprising that this has never found its way into the official recommendations.

In the meantime further work has been done at the R.A.E. to investigate the effect of sweepback and this in turn has led to a recommendation for the further modification of the criterion by the inclusion of two more parameters, the flexural-torsional stiffness ratio and the angle of sweepback. An interesting point about this is that for a given value of the torsional stiffness reduction of flexural stiffness increases the flutter speed. This indicates that it is advantageous from the flutter point of view to use as thick a skin as possible and to add only sufficient stiffening to enable the design loads to be carried. From the point of view of structural efficiency, in the load-carrying sense, this may well be far from the optimum arrangement.

The criterion in its latest published form serves as a useful guide for a preliminary estimate of the stiffness required as long as the wing is of conventional design. An era of barbed and crescent wings now appears possible, for which the value of the criterion even as a guide is rather uncertain; this means that the full-scale flutter investigations have to be completed even more rapidly than for a conventional wing if panics late in the design are to be avoided. The most lengthy part of the flutter calculation is the determination of the normal modes, a process which may well be expedited by the use of Flutter Models, although it must be borne in mind in connection with the use of these models that it is rarely possible to get the same degree of structural damping as would be obtained in the actual aircraft. The development of electronic aids to calculation promises valuable help on the theoretical side of this work and the Industry should explore the possibility of acquiring suitable equipment. It is imperative, however, that a final check of the modes be made by full-scale resonance testing and that this should be organised in such a way as to enable the results to be available almost immediately, as the tests can only be made when the prototype is almost ready to make its first flight.

Resonance tests, which have only recently been made a military requirement before a prototype first flight, have imposed on the Industry at a most difficult time a task which previously was done at the R.A.E. This task forms part of the duties of the aeroelastic group or Flutter Section and as the date of a first flight has become so important, special measures are necessary for its speedy completion. An account of how this has been done at Vickers-Armstrongs, Weybridge, is given in the Appendix.

7.1.2. *Transient stressing*

The determination of transient stresses is closely allied to flutter in that the natural vibration modes and frequencies are critical parameters. The severity of the transient stresses in flight is very largely bound up with the speed of the aircraft, the disturbing forces and the aerodynamic damping being proportional to the speed; it may be that with very

high-speed aircraft this particular aspect is not very critical. Transient stresses arising out of landing and taxying loads, however, are still a cause for concern, particularly on aircraft with overhung engine installations in which torsional and flexural oscillations of the wings and flexural oscillations of the engine mounts are bound up with the normal chassis oscillations, as well as the high frequency swing-back vibrations of the chassis in the drag plane. Installations such as wing tip tanks must also be investigated under these conditions.

7.1.3. *Aileron reversal*

Sweepback has brought the aileron reversal problem once more to the stage when it can be quite as critical as flutter from the stiffness point of view. The flexural stiffness is an important parameter and in this case the requirement is at variance with the flutter requirement, as a high flexural stiffness is desirable. It is further complicated as far as actual rolling performance is concerned by the effects of compressibility on the efficiency of the ailerons as such and on the blow-back due to deflection of the control. Investigation has shown that, in a particular case, with a wing just meeting the requirements for aileron reversal, the effect is to reduce the rolling performance at the design diving speed to one quarter of what it would be if there were no loss of efficiency of the aileron and no blow-back of the control surface. This condition occurs with a control circuit stiffness which shows a handsome margin on the specified control stiffness requirements.

If adequate rolling power is to be provided. quite an appreciable increase of stiffness, more than that necessary to meet the present reversal requirements, is necessary if the use of conventional ailerons is to be continued.

7.1.4. *Wing divergence*

This problem is not of great importance unless the field of swept-forward wings should need investigation.

7.1.5. *Longitudinal stability*

This problem is closely allied to that of aileron reversal in that it is the change of incidence arising out of structural distortion which is the critical feature. With the swept wing the loss of incidence at the wing tips, due to upward bending of the wings, leads to a forward movement of the aerodynamic centre which causes a nose-up pitching of the aircraft and hence leads to further increase of lift. There is thus a loss of stability. The effect is serious on present-day swept wings of moderate aspect ratio and thickness. Its magnitude in the future, if high aspect ratio wings with about 60° of sweep and a thickness/chord ratio of five or six per cent. become a reality, is an interesting speculation.

The wing design problems associated with high speed have been briefly indicated. How can they be overcome without considerable increases in structure weight? Some of the suggestions made for the solution of this problem are worth examining.

Dr. Williams[7] has made a number of bold suggestions. On the flutter side the suggestion was to bring the inertia axis and flexural axis forward to the quarter chord and, as he indicated, it is comparatively easy to do so with the flexural axis but the inertia axis movement is much more difficult; in fact, with the rise of compressibility effects there is a tendency to a backward rather than a forward movement of the inertia axis. The suggestion for aileron reversal was to fit two ailerons operating in the opposite sense, each operating in a definite speed range with a suitable change-over mechanism. Dr. Williams stated that model tests with such a scheme were being undertaken. It would be interesting to know the results of these tests. Another suggestion which has been made is that the reversal speed should be below the take-off speed. This implies a wing very flexible in torsion and the flutter problem would still be present; also divergence would become important. Professor Hill[8] has suggested either large chord ailerons or spoilers. It is difficult to see how the large chord ailerons would help because there would then be a smaller torsion box ahead of the aileron. Another suggestion which has been made in this direction is the variable incidence wing tip, but to get completely satisfactory results from such a scheme seems to require almost full-span tips! More satisfactory results may be obtained with a combination of variable incidence tips and conventional ailerons. There certainly is scope for the aerodynamicist in this field.

On the longitudinal stability side Professor Hill's idea of the aero-isoclinic wing is theoretically attractive. Such a wing in its best form would not undergo any change of incidence along the span under load. In practice it would be satisfactory if the

cumulative effect of the variation of incidence along the span was such as to produce no forward movement of aerodynamic centre under a positive change of the overall incidence. This may be possible if an oblique hinge is introduced in the structure at about a quarter of the span. It would not be a hinge in the mechanical sense, but rather a region of low torsional stiffness about a locally highly swept-back flexural axis. Inboard of this there would be an unswept flexural axis with high torsional stiffness and outboard a moderately swept axis with high torsional and flexural stiffness. Such an arrangement is shown diagrammatically in Fig. 24. The loss of incidence due to flexure would be compensated by the increased incidence due to rotation about the highly swept axis. If the loss of incidence due to flexure is the critical item in the aileron control problem, such an arrangement would be advantageous, while there may be some mass-balancing effect to help the flutter too. The region occupied by the "hinge" may well be the place in which to house the chassis, a two-spar differential bending structure, stiff in flexure but flexible in torsion, replacing the full torsion box which would be used inboard and outboard.

Recent investigations on flutter indicate that the introduction of controlled flexibility may enable such wings to be used with a lower overall stiffness than is possible with the more orthodox design.

7.2. Fuselage and Tail Surfaces

It is convenient to group fuselage and tail surfaces together because the aeroelastic problems with which they are associated are the same, *i.e.* elevator control and stability. Stiffness recommendations are made for fuselages and tailplanes separately but there is a clear case for considering the two together. In many cases it will probably be found profitable to provide more stiffness in the tailplanes and less in the fuselage. The only potential development seen here is towards the all-trimming tailplane with its flexural axis at the aerodynamic centre and hinged near the aerodynamic centre. In this way distortion effects will be avoided and control forces reduced to the minimum required for feel.

In the field of transient stressing, investigations have been made into the effects of the instantaneous application of elevator at high

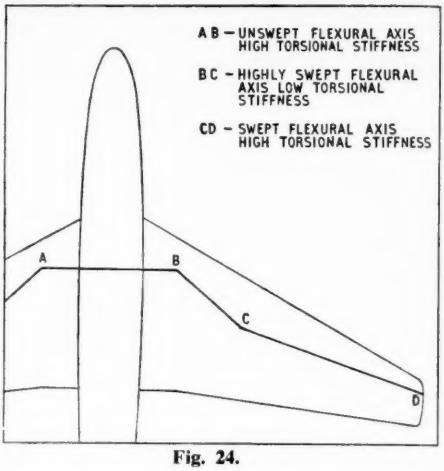

AB – UNSWEPT FLEXURAL AXIS HIGH TORSIONAL STIFFNESS
BC – HIGHLY SWEPT FLEXURAL AXIS LOW TORSIONAL STIFFNESS
CD – SWEPT FLEXURAL AXIS HIGH TORSIONAL STIFFNESS
A
B
C
D

Fig. 24.

speed with the aid of power controls and it has been shown that the flexibility of the structure allows greater elevator movements per 100 lb. of stick force, with consequent advantage to the low-speed manœuvrability.

To emphasise the importance of the whole subject it is only necessary to add that there are now very strong doubts as to whether the 30 per cent. stiffness margin at present specified is adequate to cover the uncertainties due to compressibility effects at high Mach numbers and that in the future much higher margins may have to be provided.

8. Some Immediate Problems

8.1. The Cut-out

In recently designed military aeroplanes comparatively thick skin covering is used on all main components with stiffening members usually of "Z" or "Y" section. The work on stiffened sheet in Great Britain and in the United States has made possible the design of panels suitable for the box wing or the tubular fuselage within satisfactory weight limits; it is the departure from the box or the tube which now presents the most difficult problems.

Some of these difficulties have been discussed earlier in this paper, the buried engine and undercarriage being prominent among them. The fuselage of the large military bomber presents a similar problem with its bomb bay, the presence of which seriously affects both the torsional and

flexural stiffnesses of the tube and makes the calculation of strength a problem of much greater magnitude. Cut-outs of this nature have made essential the "marriage" of the true monocoque structure to structure containing spars and longerons, thereby creating diffusion problems. The need to diffuse the load quickly is desirable from a weight point of view, but the high intensity shear flows which result show the need for stronger jointing methods. The use of high tensile steel rivets, headed cold, has proved to be one solution.

8.2. Attachment of Stiffeners

The need for alternative jointing methods to replace the present almost universal use of rivets has become more urgent with the need for heavier skins. Four methods with applications to stiffened panels are receiving attention. They are:—

(*i*) Redux

(*ii*) Continuous welding

(*iii*) Integral stiffeners formed by extruding large tubes

(*iv*) Machining complete panels from thick plate.

Each of these appears to have some advantage over riveting for the construction of laminar flow wing sections, particularly if the aerodynamicist's ideal of wing smoothness is to be achieved. Unfortunately their adoption will necessitate completely new production methods, tools and capacity, which, excluding the use of Redux, will delay their adoption for some time to come.

The value of Redux as a jointing method with comparatively thin skin has been shown by the work of the de Havilland Company. This method has won a permanent place in contemporary structural design and the use of adhesives of a similar nature warrants further development. The other methods quoted all require much more experimental investigation before any definite conclusions can be drawn.

There is an outstanding need in Great Britain for the further development of the integral stiffener. The use of thick skin and thick tapered skin, together with practical and efficient forms of stiffening, is a joint problem on which both the designer and the production engineer should now be urgently engaged.

8.3. The Thin Wing

The need for low drag in the wings of the future is being constantly declared; thin wings with thickness/chord ratios below four and no sweepback, slightly thicker wings, highly swept, and wings with variable sweepback are all possible variants of the future. At supersonic speeds it is probable that the pure shell and the solid wing will both find favour, and it is difficult to say which material will prove most satisfactory.

9. CONCLUSIONS

The problems discussed in previous sections show the need for continuous structural development. Among those quoted, the following appear to be of importance in the field of structural research.

1. An improved high tensile light alloy with more consistent transverse properties and greater freedom from distortion.
2. The production of titanium.
3. A transparency without some of the disadvantages of those now used.
4. Augmented fatigue-test programmes and the development of large test machines.
5. The general adoption of statistical instruments for recording loads in flight on civil aircraft.
6. General research into alternative methods for the avoidance of those aeroelastic difficulties which at present demand heavy weight increases to meet stiffness requirements.
7. The use of thick sheet and the development of integral stiffeners.

Acknowledgments

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Finally the author wishes to thank his Company for permission to prepare and present this paper, but would add that he accepts sole responsibility for the opinions expressed therein.

APPENDIX
RESONANCE TESTING

The high-pressure atmosphere which exists between the time a prototype is in a fit state for a resonance test and the time of the first flight has led to a development in testing technique.

This development took the form of a number of improvements in measuring and excitation technique which, together, led to a much reduced testing time and to improved accuracy.

The disadvantages of velocity or acceleration type pick-ups are their expense, the need for integration and the fact that all their calibrations are different and depend on frequency.

Since the ideal is obviously to have a pick-up at every station, connected to a selector switch, the use of velocity or acceleration pick-ups is practically impossible. A simple cantilever-type strain gauge vibration pick-up which was cheap and easy to make was therefore designed. The cross-section of the cantilever is greatly reduced near the root and strain gauges are fitted there. This gives a low stiffness to the pick-up and so allows its use on control surfaces. The position at the end of the cantilever at which the desired deflection is applied can be varied so that all pick-ups can be set to give the same calibration.

The second item that required improvement was the oscillograph. A special one was obtained* which had been developed for resonance testing. This had a high sensitivity which enabled a displacement of 0.0001 in. to be detected. A stable zero spot position allowed the use of a fixed scale on the screen. A fast return to zero after overload input eliminated the need for balancing the strain-gauge bridges on the pick-ups.

Phasing was another matter which required attention, since it had previously been just a three-way choice. An improvement was achieved by arranging to obtain a "blip" at any desired angular position of the excitor shaft. (A motor car distributor, an electric actuator, a spring, some cable and a Desynn transmitter helped.) This blip was shown on the oscilloscope screen as a small horizontal line. By adjusting the angular position of the phasing contact the blip could be moved to the top of each amplitude display as it was being measured and the phase was indicated on a Desynn receiver. The phase of each reading could be measured to within 5° by this method. Frequency control has been found to be of the greatest importance. Although the frequency may not vary sufficiently to alter the amplitude of vibration at, say, the wing tip, the relative phases may change considerably. It is therefore necessary to control the speed to at least $\pm$ 0.01 cycles/sec. to obtain consistent phase results. In practice this control was achieved by comparing the signal from a phonic wheel on the excitor shaft with the output of a beat frequency oscillator on a double beam oscilloscope. Any variations could be corrected manually.

With these improvements the testing time on a large machine using 48 pick-ups, was 18 hours and by the end of this time all results were plotted.

*From de Havilland Propellers Vibration Division.

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DISCUSSION

Dr. H. Roxbee Cox (Chief Scientist, Ministry of Fuel and Power, *Fellow*): The list of problems which Mr. Gardner had given at the end represented a research programme which really ought to be in hand.

Mr. Gardner had said that no optimum structural design had emerged; no royal road had been discovered for designing very light structures. There had been a number of clever attempts, and he hoped that people

would go on trying and thinking of novel constructional systems.

He was glad the printed paper had referred to the contribution which the Society had made through the Structures Data Sheets. The data sheets were a great help to designers and he thought it should be more generally known that they all sprang from a suggestion made by B. S. Shenstone in 1940.

Mr. Gardner had mentioned the importance of structural efficiency and weight saving, and had pointed out that a saving of 2 per cent. of the structural weight, from say 30 to 28 per cent. would make a saving in take-off weight of no less than 6 per cent. In the spoken lecture he indicated that that was the first approximation. In point of fact if they approached the problem in a rather more academic way, there were possibilities of saving three or four times what Mr. Gardner had mentioned as being possible. There had always been the possibility of refining design to produce that sort of effect, but Titanium was a material which ought to provide one or two more per cent. structure weight saving than they could obtain in any other way. It was of vital importance, but it would have its manufacturing troubles. The unusual ratio of Young's modulus to ultimate tensile strength might cause structures made with it to be a little different from those they had been used to in the past. The material, although strong for its weight, had a relatively low "*E*".

He supposed an aeroelastic problem was one in which the aerodynamic and elastic actions were interdependent. If they took that definition it was difficult to find a problem which was *not* aeroelastic. But they could approximate quite sensibly in solving many problems by assuming rigidity for the structure.

There were many problems however in which they could not neglect the interdependence. The aerodynamic loading on a wing was itself dependent on the distortion of the wing, and he would be surprised to learn that for stressing in these days they could assume the wing to be rigid. They had to find, presumably by successive approximation, the aerodynamic load system corresponding to the distortion it produced.

That must be quite a serious calculation in a good many wings today.

Mr. Gardner turned down the proposal for having a large chord aileron because there would be a smaller torsion box ahead of the aileron. He was not sure that the suggestion could be so lightly dismissed. Nor could the old idea that if an aileron at the back made the wing twist the wrong way, the thing to do was to put the aileron at the front and make the wing twist the right way ! Perhaps Mr. Gardner would comment on that.

Mr. Gardner had paid a pleasant tribute to the work done at the R.A.E. in the field of aeroelasticity and it was interesting that the three people who years ago had initiated the R.A.E. aeroelasticity work were present that evening—Dr. Pugsley, Dr. Williams and himself. It was extraordinary to find after 20 years that the same problems were still causing trouble.

Mr. Gardner had referred to the far off times when one bothered about the number of members and joints in a frame and attributed the basic theorem to Castigliano; he could assure Mr. Gardner it was Clerk Maxwell. Castigliano was responsible for two strain energy theorems. If those theorems were no longer used he congratulated structural designers because they had caused endless trouble.

Dr. Williams (Royal Aircraft Establishment, *Fellow*): He had been intrigued by Mr. Gardner's mention of a method of solving a problem which he called solving by brute force. Would he expatiate a little more fully on that method ?

Discussing pressurisation problems Mr. Gardner had described the substitution of a concave for a convex bulkhead, and had said that when designing for minimum weight the arms stretching fanwise from the centre of the bulkhead were designed to be loaded in "compression only," the plating in between taking tension only. Surely minimum weight was to be obtained by way of making the individual members do double duty, rather than by making each perform a single function ?

He wished to defend (since he had something to do with its inception) the present criterion for fatigue that aimed at two million repetitions of a mean load of 1 *g* plus an alternating load of 7½ per cent. of the ultimate.

Mr. Gardner said it was too severe and could not be achieved without great care in the initial design and a high standard of workmanship. After all that was the purpose of the criterion, and since it was not mandatory it could be interpreted in the spirit rather than in the letter.

He deprecated the custom of calling the cumulative damage law a law; hypothesis if they liked, but not law. When Mr. Gardner said, "Its rigid application must be regarded with considerable suspicion," he was, he thought, putting it mildly.

He agreed that the easiest, and in some cases the only, way of finding the normal modes of flutter was by means of a scale model. Such a model should be a very simple one and simplification should be (as described in a recent paper of his*) by way of a completely separate representation of the inertia forces, thus greatly facilitating the correct representation of the stiffness properties.

Structural damping he did not think very important since, being small, it could not affect the modes very much.

Mr. Gardner's surmise that at very high speeds transient loading might not be very critical was a little optimistic. The damping forces certainly increased with speed (not, by the way, with the square of the speed as mentioned in the paper), but so did the sharpness of the gust, and the final result was that, although the percentage overswing above the static deflection fell off at very high speeds, the absolute value of the stresses produced went on increasing and therefore were likely to remain critical.

With regard to locating the various wing axes in favourable positions to prevent flutter, it had been pointed out that the inertia axis was much more obdurate than the flexural axis in that respect. No doubt that was true if one tried to bunch the two together at the quarter chord, but much could be done without quite reaching that ideal so long as the inertia axis was ahead of the flexural.

The twin aileron scheme he had once suggested for lateral control at high speeds had been tried out in model form. It was found that so much torsional flexibility was required over the outer part of the wing in order to make the outer aileron reverse its control at an early enough speed that they got virtually a rotating wing tip. It seemed clear from those experiments that the next step should be to retain the inner aileron and have an elastically restrained rotating wing tip with a reversal speed about twice the stalling speed. Above that speed of minimum control, as well as below, they would then have a control power progressively increasing in either direction.

He had had some connection with Professor Hill's aero-isoclinic wing and, according to his investigations, it seemed not only theoretically attractive, but attainable in practice. It achieved its object by exploiting a comparatively low torsional stiffness extending from root to tip, with a flexural axis situated well aft.

From his experience in that work he would say that the kind of arrangement sketched by Mr. Gardner in Fig. 24 was just not feasible, because any restoration of incidence achieved by *twisting* about the swept-back portion BC of the flexural axis was likely to be lost again by the extra loss of incidence induced by the *bending* of that very same swept-back portion.

Professor R. L. Lickley (College of Aeronautics, *Fellow*): The design of joints was an important factor structurally and more research might be done on the development of more efficient joints.

Although the lecturer had shown some steel castings he had said nothing about future use of high tensile steel castings. Did he think there was a future for them and for high tensile light alloy castings? Also, did he think they had missed the effect of elongation in directional leading problems?

With regard to titanium he could only endorse what Mr. Gardner had said about it and point out that an American publication recently mentioned that 4,000 tons of titanium were to be manufactured next year. Probably if they had as many ounces in Great Britain they would be doing well. They should be doing far more work with titanium and its alloys.

Tests at the R.A.E. on the fatigue of drive fitted bolts had been mentioned. Certain work done at the College of Aeronautics on static testing of lugs showed that with fitted bolts there was less stress concentration with a given load than with holes which were about 10/1,000″ oversize. That would make the fatigue results quite easy to understand in that the stress concentration would tend to decrease the fatigue life.

Like Dr. Williams he had been surprised that Mr. Gardner had mentioned the cumulative damage law. He believed the best that could be said about it was that it was an unconfirmed hypothesis.

R. C. Morgan (British European Airways, *Fellow*): British European Airways had

*Note on the Distortion Characteristics of Swept and Cranked Wings in Relation to Flutter and Other Aeroelastic Phenomena, D. Williams, *Journal of the R.Ae.S.*, February 1951.

made a start towards the provision of more statistical data to assist the structural engineer. *V-g* recorders had been installed in the Viking fleet for two years, but since they did not provide information on the frequency of loads encountered in flight, counting accelerometers were being introduced in their stead.

He thought that if the operator wanted the savings in weight which would produce increased revenue of the order indicated by Mr. Gardner, the operator must be prepared to provide means of obtaining the data which would enable such savings to be effected.

Mr. Gardner had taken the example of a civil aircraft of 50,000 lb. all-up-weight. A breakdown of this figure on the average present-day figures might give a disposable load of 20,000 lb. and an equipped weight of 30,000 lb., the equipped weight comprising 15,000 lb. of structure and 15,000 lb. of engines, radio, instrumentation, and so on. The two per cent. saving in all-up-weight mentioned as a target, therefore, would actually be a saving of 1,000 lb. in the 15,000 lb. of structural weight—or a saving of 6 per cent.

If the target suggested in the paper could be advanced by the structural engineer on his portion of the equipped weight, how much more should a similar project be undertaken on the remaining 15,000 lb. of equipped weight.

R. S. Stafford (Handley Page Ltd., *Fellow*): Some work on fatigue strength with oversize bolts was done at the Battelle Memorial Institution, U.S.A., which led to the conclusion that with steel bolts in light alloy members, "after almost any load" the bolts assumed normal fit and there was no apparent difference in fatigue life. It would be interesting to know what were the factors which led to the different conclusion with Fisher's work.

The improvement in fatigue life of certain structural elements caused by over-stressing was significant, but he hoped Mr. Gardner's remarks would not inspire pilots to indulge in violent manoeuvres or early flights in the hope that that would improve fatigue life. When a customer took delivery of a new aeroplane, he expected bolts, pins, and so on, to have good fit and unless they were clever with the over-stressing technique, some care would need to be exercised in tightening up of bolts after such flights.

Written Contribution: To be historically accurate, he would suggest that a fourth item should be enumerated, namely the design of light ribs. They used to be greatly concerned about the effects of pulsating fabric loads and it was interesting that that was probably the earliest example of the Industry's practical appreciation of fatigue as an important design feature.

He did not agree altogether with the suggestion that heavy spar booms were necessary even with buried engines. With an aeroplane which they had in an advanced state of construction, and in which the engines were buried, they had found it practicable to reduce spar boom areas to very small proportions by the use of a corrugated panel construction.

He was a little puzzled by Mr. Gardner's conclusion that it was impractical to design for fuel with Reid vapour pressures of 7 lb./in.^{2} without fuel loss. When the conditions obtaining at the base or side walls of an integral or bag tank were considered, normal accelerations, rolling and centrifugal effects might well give rise to total pressures exceeding 20 lb./in.^{2}, so that a difference in inflation pressure of 3 or 7 lb./in.^{2} became of secondary importance.

They had found that by use of one-inch thick corrugated wing panels with closely spaced spar webs, a tank inflation pressure of 7 or 8 lb./in.^{2} presented no undue difficulty and involved a negligible structure weight penalty.

He thought a word of warning was necessary about transient stresses in swept wings; while it was true that for a straight swept wing the adverse effects of over-swing were offset by the aerodynamic twist associated with flexure, in the case of a kinked and swept wing, a gust would excite the torsional mode giving rise to shear stresses appreciably greater than for the same gust acting on a rigid wing.

Again, care must be exercised in adopting the oblique hinge portrayed in Fig. 24, since the change of incidence due to flexure which this device sought to avoid was an important relief in the gust case. It seemed, therefore, that while longitudinal stability might be improved in the manner suggested, the improvement might be accompanied by considerable cost in structure weight.

With power controls they had a ready means of reducing blow back to small proportions in locating power units close to the control surfaces so that interconnecting

mechanism was short and had high stiffness at low weight.

With regard to Mr. Gardner's remarks on the development of wing panels for aerodynamically smooth exterior, they had developed a satisfactory panel using corrugated sandwich with spanwise corrugations. The outer skin was spot welded to the corrugation while the inner skin was blind riveted. Model tests at Reynolds number 10×10^6 had shown that by suitable choice of welding electrode, the very slight surface indentations did not affect transition. Another development they had followed was metal honeycomb sandwich, but while this satisfied all their requirements for smoothness, strength, stiffness and ability to withstand repeat loads, they were unable to find an adhesive with the ability to form a meniscus during the curing process and at the same time being resistant to deterioration under conditions of high temperature and humidity. The development of a suitable adhesive for this application was well worthwhile. He agreed wholeheartedly with Mr. Gardner's plea for more structural development and research.

F. H. Pollicutt (Folland Aircraft Ltd., *Fellow*): He had had the impression for some years that structural research had received much less attention than aerodynamic and electrical research; all three subjects should be pursued with equal vigour and determination and with one notable exception, they could do much worse than follow Mr. Gardner's plan. The exception was that Mr. Gardner, by inference, was satisfied with aluminium alloys as his major structural material. Before they could make any marked difference to structural efficiency at present they would have to have an alloy of greatly increased stiffness compared with its other mechanical properties. He thought that for certain types of structure such materials as asbestos and glass plastic would have to fit into the picture.

He disagreed with Mr. Gardner's statement that "pressurisation has continued to remain an intractable problem to the present day." He had been impressed by the way in which designers throughout the country had produced pressurised cabins with very small weight penalties. He believed the factor of 2.0 on the working pressure was inadequate for the satisfactory design of plating joints. Apart from that, test work and experience did not suggest that they should look forward inevitably to a period of trouble in service. The word "intractable" in the paragraphs to which he had alluded meant "unmanageable" and he thought Mr. Gardner did not mean that at all.

H. B. Howard (Ministry of Supply, *Fellow*): He had been glad to hear Mr. Morgan's reference to the R.A.E. recording accelerometer. The point there was that the *V-g* recorder did not give the figures wanted for fatigue and the recording accelerometer was developed by the R.A.E. primarily to overcome that objection. The Ministry of Supply were having a number made which would start coming off the line, he believed, in 1952. Plenty would be available, and for operators who would like to have them on their aircraft, the Ministry would be glad to supply instruments and instructions on how to use them.

It seemed to him that the double skin with corrugated web between the two skins was promising for modern aircraft. It gave smooth internal surfaces and plenty of room inside the wing and ribs could be cut down or even eliminated. He would like Mr. Gardner's views on that.

G. H. Lee (Handley Page Ltd., *Fellow*): Mr. Gardner's remark that upward bending of the wing tips caused reduced incidence there and so reduced stability was true, but it needed a little amplification, in that while this phenomenon inevitably reduced the manoeuvre margin and caused trouble in the pull-out and similar cases, it was possible, actually to get an improvement in the static stability due to other aeroelastic effects. The static margin, K_n, could be increased due to aeroelastic effects, and one way in which that could happen was that with a cambered wing increasing speed would cause the C_{m_0} due to camber to twist the wing's leading-edge down at the tips. Thus, on that account, increased speed would give an increased wash-out, which would go some way to offset the upwards bending effects.

Another way in which it was possible to modify the de-stabilising effect of bending was simply to have the flexural axis reasonably far back on the wing. Increasing lift gave a twist which would tend to wash-in at the tip as the load was put on, and that tended to cancel out the bending-up effect, and so went part of the way towards the isoclinic wing. That was another version of the rather complicated flexural axis variation.

With regard to the question of tailplanes,

if there were sweepback there would still be upwards bending at the tip, even with the flexural axis coinciding with the locus of aerodynamic centres, and consequently there would still be a loss of tailplane $dC_L/d\alpha$. That could be attacked in the same way as had been suggested for the wing, *i.e.* shift the flexural axis rearwards so that torsion tended to cancel out bending.

It had also been suggested that putting the aerodynamic mean centre of the tailplane (if it were all-moving) very near the hinge line (permissible with power controls) would cut down hinge moments. That was true, but due to Mach number variation and to aeroelastic effects, the aerodynamic mean centre would vary quite appreciably and so hinge moment could not by any means be eliminated; over-balance for some flight conditions might have to be accepted.

He could not understand the point made in Section 7.2 where the author stated: ". . . flexibility of the structure allows greater elevator movements per 100 lb. of stick force, with consequent advantage to the low-speed manoeuvrability." Could Mr. Gardner clarify that ?

R. Tatham (Saunders-Roe Ltd., *Associate Fellow*): He thought the problem of temperature gradient in relation to the strength of materials should have been mentioned. He had come across cases where more or less unaccountable failures were occurring and the only possible explanation at present seemed to be the existence of a permanent temperature gradient over a small distance. There, he thought, the stresses set up were of such a size that failure was occurring unexpectedly.

He had been interested in the reference to the method of analysis which had led to the adoption of the elliptical port lights in pressurised cabins. Some work had been done at the R.A.E. in which consideration was given not only to the stress conditions arising from pressurisation, but also to other stress conditions. What worried him about those elliptical holes, was that, while it was true that they would get only tension in the boundary member under stress conditions corresponding to pressurisation alone, the addition of other stresses, such as shears in the skin, meant that the condition of only tension in the boundary member would no longer exist. Had anything been done to take account of that ?

Another point which had always seemed to him to be an awkward one was that they worked up to the ultimate tensile strength of the material and continued to apply elastic theory. Beyond the elastic limit stress was not proportional to strain any more. It was possible to take account of the actual stress strain diagram itself, by use of form factors in some cases for particular components. But if they took advantage of the increase that they could get by using the actual stress-strain diagram they might gain so much that they would no longer automatically satisfy proof conditions. That required investigation.

E. D. Keen (Sir W. G. Armstrong Whitworth Aircraft Ltd., *Fellow*): As a structural engineer, it was heartbreaking, bearing in mind the tremendous amount of work done to make the basic structure more efficient, to see how things grew on aeroplanes. An appeal should be made to all those responsible for adding refinements to cut them down as much as possible. Much good work had been done, he believed, by the S.B.A.C. Weights Committee to control the weights of components, but why not leave many of them of altogether ?. There was too much complication on modern aeroplanes.

Another way in which structural engineers were at a disadvantage was in the design of radomes. Here, the primary consideration appeared to be electrical efficiency regardless of the resulting structural qualities. He understood that the optimum electrical performance was usually given by a sandwich type of dielectric material. Although attractive at first sight, such constructions involved impracticably thin face materials, subject to accidental damage and rain erosion. A thick, solid material might well be less acceptable electrically but the structural difficulties would be greatly alleviated. A much closer liaison was required between the electronic and the structural designers involving frank and open discussion of one another's problems so that a compromise could be worked out.

More work should be done on repeated loading tests of fuselages. His firm had concluded a run of 10,000 cycles on an Apollo fuselage without significant failure. Not one of the windows had to be changed throughout the run and that fact alone was encouraging. Further tests on the same specimen were in progress.

He was not sure whether the now generally accepted method of holding laminated glass

panels by the projecting inter-layer did not result in a thicker windscreen pillar, which detracted from the all-important view through the windscreen and side panels.

With regard to repeated loading tests on joints the results given by Mr. Gardner on the effects of interference fits between the pin and the hole were very interesting. Could the same effects be produced by using tapered split collets with parallel centre bolts? He had in mind an apparently successful design that incorporated tapered bushes of this sort which had obvious advantages when applied to transport joints. He thought that repeated loading problems were of less significance in the design of a fighter with an ultimate factor of, say, 11 than that of a civil aeroplane with an ultimate factor of 4 or 5.

Machined and extruded wings had been mentioned as being logical lines of development. It was most important that work should go on in this field. Unfortunately it was difficult to justify a research programme at present on the grounds of expense. Large machine tools were required but the technique was well-known. He thought that those responsible were apt to be put off by the fact that it was not possible to prove on paper immediately that a machined wing could be made as cheaply as one of existing construction. That attitude was entirely wrong. Firms should try it out and if necessary find someone who would defray the initial cost.

As a member of the Structures Committee of the Society, he could reassure members that it was very much alive.

Professor A. G. Pugsley (University of Bristol, *Fellow*): Mr. Gardner had touched on the development of "flutter sections" in aircraft firms. It might not be generally realised that it had taken 20 years to reach that stage. Having regard to the relatively rapid development of "stress sections," this was really rather surprising. He wondered whether it was due to the inadequacies of himself and others in the early stages, or to neglect due to a tendency to treat flutter and the like as mathematical mysteries only to be dealt with by high-brows at Farnborough and the N.P.L. He was glad to know that the position was changing rapidly.

The exponents of resonance testing were commonly concerned these days with high overtones among the natural frequencies of aircraft structures. It was fairly common to find that calculations predicting such frequencies and modes differed considerably from resonance tests results. Moreover it was also fairly common in the experience of anyone doing resonance testing, to discover that those higher modes that were being measured were not strictly natural modes at all, there being large differences of phase between the movements of various parts of the structure.

Would Mr. Gardner comment on that? Perhaps they had ignored too long these phase differences, which might be due to structural damping.

If one looked back through the history of structural work in general it was fair to say that the biggest gains from new materials and the biggest movements towards new materials nearly always occurred when a major change of structural arrangement was being envisaged. The classic example in aeronautics was the change from biplane to monoplane; that gave a chance for the new materials of the day in a way that the biplane never did. Today they seemed on the brink of similar changes, either in relation to the thin wings of supersonic aeroplanes or to delta types and the like, and now was a good time to explore to the full the possibilities of new materials.

Mr. Gardner and a number of speakers had been depressed at the apparently small volume of structural research in the aeronautical field. He felt more hopeful. The bulk of structural research of a high type in this century had been the work of people primarily concerned with aeronautics; and very little structural research had gone on in other fields of engineering. That was not so today. The volume of structural research outside the aeronautical world was very many times greater than ever before; and it was appropriate in the building of the Institution of Civil Engineers to draw attention to this fact and to hope that the two fields of research would get together as frequently as possible and benefit from joint discussions.

H. A. Mettam (Royal Aircraft Establishment) *contributed*: A considerable amount of research work was already in progress on the problems to which Mr. Gardner had drawn attention. Mr. Gardner, for a long time, had been one of the representatives of the S.B.A.C. on those committees which co-ordinated much of the structural research done by the official establishments and by various firms in the Aircraft Industry. The only point on which disagreement ever

occurred was the amount of the available effort which should be directed towards any particular problem. Members of the official establishments had made, and were continuing to make, valuable contributions to the development of new methods of calculating the strength of novel types of structure.

Mr. Gardner referred to W. A. P. Fisher's work on the importance of a tight fit on bolts, loaded in shear, for increasing the resistance to fatigue. The same author had drawn attention to the importance, for the same purpose, of pre-tensioning bolts, loaded in tension. It might be of interest to comment on the probable effects of temperature in both those cases. When flying at altitudes, and not flying fast, the internal joints would be colder than at ground level. Steel bolts in light alloy fittings would then fit more tightly and fatigue of joints with bolts in shear would be improved. With steel bolts in tension through light alloy fittings however, the improvements due to pre-tensioning would tend to disappear. Under really high speed flight conditions on the other hand, the temperature rises described by N. J. Hoff and mentioned by Mr. Gardner would improve the tension case and worsen the shear case.

Mr. Gardner referred to the improvement in the fatigue properties of a metal aircraft structure due to loading up to 2 *g* during early flight tests. An interesting parallel occurred in the case of transparent cockpit hoods, which were improved in ultimate strength by proof loading under pressure at elevated temperatures. Both those observed effects appeared to be due to slight plastic distortions relieving high local stresses.

Mr. Keen had suggested that split taper bushes should be used. Tests done in 1943 by one of the aircraft firms on a particular type of joint using such bushes showed the importance of correct orientation of the split. The strength of the joints was about 10 per cent. greater with the slots placed at right angles to the direction of the load than when they were placed along it. Such a reduction would not necessarily apply to all design of joints using split bushes, but a warning appeared to be desirable.

W. A. P. Fisher (Royal Aircraft Establishment) *contributed*: To modulate the somewhat pessimistic note sounded by Mr. Gardner about the fatigue strength of joints, a brief review of the results of recent researches at R.A.E. might be interesting.

The figure of 4 to 1 increase in endurance quoted by Mr. Gardner for a loaded hole when the pin had a high interference fit erred on the side of caution.

At a mean stress (on net area through the hole) of 6 tons/in.^{2} and an alternating stress of 1.8 tons/in.^{2}, push-fit bolts lasted for about 200,000 cycles. For various degrees of interference fits up to 0.003 in. there was only a slight improvement. But for interference fits just exceeding 0.003 in. specimens remained unbroken after *10 million* cycles. This represented an increase in endurance of *more than fifty times*.

Thus, although the loaded holes tested at Cranfield and the N.P.L. gave a fatigue strength of only one ton/in.^{2} for 2 million cycles, the R.A.E. tests for very tight fits indicated a fatigue strength greater than 1.8 tons/in.^{2} and that for a much higher mean stress. Some specimens, in fact, failed not at the hole but in the plain portion, as a consequence of fretting on the flat surface. Those results, although of little comfort with respect to aircraft already built and flying, nevertheless showed important possibilities in connection with new designs.

Mr. Stafford had quoted the results of some American work to show that interference fits gave no improvement in fatigue strength. But the American tests were confined to interference fits not greater than 0.0015 in., and up to this degree of interference the R.A.E. results also showed no appreciable effect.

Seeing that the loads required for assembling and extracting pins with such high interference fits would be excessive in large joints, further experiments made recently at R.A.E. were of interest. In those, mild steel bushes with various degrees of interference fit were used, and similar improvements were found to those obtained in the tests without bushes. The pins, being only a push fit, could be assembled easily in the bushed holes.

In multiple-bolt joints, the difficulties of fitting were further increased. For such joints, it had been found that tight clamping could give an increase of endurance as much as 9 to one. Thus two basic principles for overcoming the apparent low fatigue strength of joints were available to the designer, with practically no weight penalty:

(*i*) high interference fits in loaded holes,

(*ii*) tight clamping of faying surfaces.

The application of those principles to existing aircraft joints had already given valuable results.

MR. GARDNER

Dr. Roxbee Cox: He had been intrigued with the statement about the origin of the Structures Committee as it was now known. He well remembered Dr. Roxbee Cox was the first chairman of the Committee. It was rather pleasant to know that Mr. Shenstone originated the idea. He hoped that more work would be produced by the committee in the near future.

On the question of structural weights he had had in mind one of the papers given by Dr. Roxbee Cox; he thought the theory of successive approximation most fascinating and that it could be carried to great lengths, but as with all work in industry there was a factor which was probably the biggest problem of all for structural engineers, the time factor. It was probably the one which dictated very definitely how much work could be done, how much structural development was possible and how far they could go in improving structural efficiency. If more time were available greater improvements would be possible.

The low value of Young's modulus was one of the disappointing features of titanium, but it was clear from work already done in America that big structural gains were still possible. They should not sit back in Great Britain and let other people do the work.

He agreed with Dr. Roxbee Cox on aeroelasticity. He was indeed honoured to see the three "grand old men" of this subject assembled together. It was quite true that as a result of that work Great Britain had been leading the world in the examination of structural problems concerning aeroelasticity. He felt that the rigid wing had now had its day. The basic problem was the successive approximation necessary to find out how the aerodynamic loads changed with the structural deflection.

On the large chord aileron he was probably more impressed by the difficulty of getting the necessary stiffness at the very small section at the leading edge made essential by that assumption. He did not feel that they were anywhere near the solution of that problem yet. The only possibility he could see was the one that they had tried many times to achieve—the all-moving wing—but that, he thought, meant even greater mechanical difficulties.

Many had considered at great length the aileron at the front, the most successful form being the slat.

On the point of disagreement he stood corrected. The theorem, he believed, was not strictly Castigliano's or Clerk Maxwell's, but was closely connected with both.

Dr. Williams: The brute force method of solution was one which was used too often. It was one in which a part was drawn—overstrength—and then made to work, irrespective of whether that was the best way of doing it. It was a method sometimes used when time was short.

The bending of the bulkheads he would like to argue at more length. He did not agree, but he certainly agreed with many of Dr. Williams' remarks about fatigue defined by the value of 1 *g*.

On transient loads he believed the point about high speed was right, but as Dr. Williams had been one of the original workers in the field of transient stressing it was a matter on which he would hesitate to cross swords with him.

The bi-aileron scheme was one which was a matter of opinion. Two ailerons could be used—he believed had been used. He knew there were schemes in existence, but it was difficult to explain the theory in a few words.

He was very glad to hear Dr. Williams say that the aeroelasticity theory was too difficult for him. He agreed entirely on this point.

He had been interested and somewhat surprised to hear Dr. Williams say that the figure shown in Fig. 24 was not possible. He had been quoting from actual results. The relationship between the loss of incidence due to bending and the gain due to twist clearly depended on the relative stiffnesses.

Professor Lickley: The question of structural joints, he believed, was one of the major detail design problems in the Industry; a good deal more research work was wanted. The question of getting two pieces of material in the same place was always one which was not often solved properly, often because of the lack of time. A serious exploration of that particular field would be well repaid.

He agreed that new steel casting specifications had a promising future. He would be interested to hear why the large castings (Figs. 12 and 13) were not popular.

The importance of elongation he had touched on in the lecture. He believed the very low transverse elongation of some of the more modern materials was perhaps the chief contributory factor in those difficulties

experienced with directional loading. If only metallurgists could improve the low elongation in the transverse direction he believed they would be materially improving other properties of some of those alloys. He would like to see transverse elongation quoted in the specification.

The production of 4,000 tons of titanium in America was a thing of which one heard with considerable envy. He hoped that it might be possible for some of that material to be made available in this country.

Undoubtedly the interference fit was one which would play an important part in the reduction of stresses. After all most fatigue failures were due to high stress concentration. He would like to see more work done in the determination of the reduction of the peak stress concentrations due to the various fits.

Perhaps he had been a little too brief in the paper on the use of the accumulative damage law, and he apologised. He agreed, however, with both speakers on this point.

Mr. Morgan: He had been most pleased to have Mr. Morgan's comments on the importance of more statistical information and to hear his challenge to the people responsible for the weight of aircraft equipment. Aircraft designers were most conscious of the latter point and would like to see the problem considered, both for weight reduction and the elimination of unnecessarily elaborate equipment.

Mr. Stafford: The American work on fitted bolts, he believed, had not been carried sufficiently far. The work described in the paper had shown that for interference fits below $2\frac{1}{2}$ thousandths of an inch a bolt of 0.75 diameter gave no material improvement. He suspected that the tests at the Battelle Memorial Institution had not carried the interference fits sufficiently far. He hoped that this feature, as far as joint design was concerned, would become the subject of urgent research.

On the question of $2g$ loads during the test flights of production aircraft, he stood by his remarks in the paper as he believed them to be literally true. As far as ultimate safety was concerned, however, he believed that Mr. Stafford was right about the advisability of tightening up bolts after such flights.

He agreed that the design of light ribs was a problem of the biplane period, in fact an unusual one, as the majority of that type of rib design was usually found, on test, to be greatly overstrength.

On the subject of the impracticability of designing for fuels with high Reid vapour pressures, the penalty of design for an internal pressure of 7 lb./in.^{2} without loss of fuel admittedly would vary with different basic designs. If adjacent structure had been designed to cover pressures exceeding 20 lb./in.^{2} it was clear that a very different type of structure was envisaged. It was important, however, to note that the weight increase in mind when writing the paper affected mainly the top surface of the wing, which was not affected by rolling and centrifugal effects to the same extent.

He appreciated the point about the arrangement shown in Fig. 24. The importance of the potential loss of gust relief clearly depended upon the extent to which the gust condition designed the wing structure. It was agreed, however, that the blow back of control surfaces could certainly be kept to a minimum by locating power units as close to the control surface as possible, although in the case of ailerons that was not always the most practical arrangement.

Mr. Pollicutt: On the point of disagreement he could not do less than quote the original paper which said that the problem of pressurisation continued to remain more or less intractable to the present day. That designers of pressure cabins had achieved a high measure of success was undeniable. He considered that many of the associated problems mentioned in the paper were still awaiting completely satisfactory solutions.

Mr. Howard: The point about double skin introduced automatically considerations of sandwich construction. The double skin connected by a corrugated skin as interspacer was clearly a satisfactory solution in many designs. The inner skin should be as thin as possible as, particularly with the more modern wing with a low thickness chord ratio, it was an advantage to keep the outer skin as thick as possible.

Mr. Lee: He agreed with Mr. Lee's comments on static margin and manoeuvre, and felt that his comments regarding tailplanes could produce a long and involved argument.

In answer to the question on Section 7.2 about the effect of structural flexibility on permissible elevator movement for a stick force of 100 lb., this was affected by the type of "feel" unit adopted. The design requirement with the simple "Q" feel unit adopted was such that a stick force of at least 100 lb. could be applied in the most adverse

conditions without breaking the aeroplane. As calculated on the basis of a rigid structure and with the feel unit stiffness designed to cover the stick force per "g" requirement for medium and low speed conditions, excessively high tail loads could arise in the high speed condition due to instantaneous application of elevator. Dynamic calculations which took account of the flexibility of the rear fuselage and tailplane showed, however, that those high loads did not occur. If, therefore, the structure had been treated as rigid it would have been necessary to increase the design strengths of both tailplane and rear fuselage, or alternatively to have increased the stick force per "g" at low speeds.

Mr. Tatham: The remarks on temperature gradient were fully justified. That problem had not yet been considered sufficiently; also, he admitted that in describing the theoretical reasons for adopting the elliptical opening he had deliberately evaded the secondary stresses mentioned by Mr. Tatham. That was undoubtedly a complication, but compared with pressurisation it was a secondary problem which should be considered in the early design stage.

As to plastic flow, he agreed that the form factor was an extremely useful means of improving structural efficiency in terms of normal static strength but he had come to the conclusion that the use of the form factor in the consideration of fatigue was one which should be approached from a different point of view. He was not sure that the use of form factors was a wise procedure when fatigue was the first consideration.

Mr. Keen: He endorsed Mr. Keen's remarks on radomes. The matter was one which certainly should be given much more consideration. The plea for simplification by elimination was one which could not be made too often. The results of the repeated loading tests on the Apollo fuselage were most encouraging, and Mr. Keen's remarks on some effects of the generally adopted method of holding laminated glass panels were certainly important.

He certainly believed that tapered collets could be used with advantage. Taper pins had proved to be infinitely better than the standard bolt or parallel pin when viewed in the light of the results of recent fatigue investigations. From experience of work completed at Weybridge he would imagine that the use of tapered collets should materially improve fatigue strength, provided that sufficient care was taken in the method of introduction.

Professor Pugsley: Professor Pugsley's comments on flutter were always extremely welcome and it was pleasant to report that the mysteries associated with flutter, at least as far as one firm was concerned, were receiving every consideration.

In resonance testing the more fundamental modes still seemed to be of greater importance than the high overtones among the natural frequencies, particularly of the wing structure. It was true that the latter were not strictly natural modes and that large phase differences occurred. However, the agreement between calculated and measured frequencies had been fairly reassuring in recent aircraft, and he felt that, provided structures remained reasonably conventional, there was no reason why that agreement should not increase.

On the question of new material he believed that once again Professor Pugsley had made one of his profound statements. He thought that prophecy might well be borne out in the new generation of aircraft soon to be designed. The news of the volume of structural research outside the aeronautical world was most reassuring and he hoped that the results would become available to the Aeronautical Industry.

Mr. Mettam: He had read Mr. Mettam's contribution with a great deal of pleasure. He agreed entirely with the points made and wished to acknowledge and express his thanks for Mr. Mettam's constructive criticism over a long period of years in connection with his official position in the R.A.E.

Mr. Fisher: Mr. Fisher's work on fatigue was considered to be of the utmost importance. The results of the interference fit were so new when the paper was written that the actual figures were deliberately written down in the belief that experience would show that the scatter of results in that field would be similar to the scatter experienced with similar forms of stress concentration. It was agreed, however, that on the results of the work described a much more optimistic statement could have been made. It was pleasing to see the comparison of the Farnborough results with the work of the Battelle Memorial Institution.

The need for more exploration into the effect of tight clamping in aircraft joints was admitted and he hoped Mr. Fisher would be able to continue his good work in that field.

A Method of Estimating the Loading on an Elastic Airframe

B. A. HUNN, B.Sc.

INTRODUCTION

The behaviour of an elastic aircraft in flight may be divided into dynamic and static cases with an intermediate class of phenomena under the heading of stability. The dynamic category is concerned with flutter and divergence effects and since these are destructive when they occur they are naturally given precedence in the aeroelastic analysis of an aircraft.

Having eliminated these dynamic effects by suitable design it is natural to turn to the static case which assesses the manoeuvrability of the aircraft. With the advent of sweepback and the attainment of higher Mach numbers it becomes increasingly difficult to determine the behaviour of modern aircraft even when regarded as rigid. The inadequacy of the lifting-line theory for swept-back wings and the inconsistency of wind tunnel tests, owing to the arbitrary nature of Reynolds number corrections, have led to the development of a lifting-surface theory which has had a fair measure of success. The test of any theory is not only its ability to predict results but also the ease with which it can be used from the computers' point of view. This theory has been elegantly applied by H. Multhopp[1] to the calculation of derivatives and his method has been subsequently used by D. Küchemann[2] to produce a method which reduces the computing labour to a negligible minimum.

It is a common practice in aeroelasticity to apply the results of such calculations on the hypothetically rigid aircraft to their practical and deformable counterparts. For example, a spanwise distribution of lift derivative given for a rigid wing may be applied to the same wing when distorted and, by suitably modifying the local lift by means of the local incidence, an overall lift is deduced. This automatically excludes the possibility of local incidence change propagating its influence elsewhere along the span by aerodynamic means. In other words, although an initial three-dimensional solution of the lift is used it is subjected to a two-dimensional treatment in the elastic problem. While very stiff wings are being considered this approximates to the correct solution, for by virtue of the large stiffness the lift loss in any event will be small, so that, say, a 50 per cent. inaccuracy in the estimate of the lift loss will be a very small inaccuracy in relation to the overall lift. However, in general, this method may give an answer which deviates considerably from the truth.

An analogous problem is the determination of the coefficient of damping in roll. The aerodynamicist can show that the symmetric and anti-symmetric distributions of lift derivatives differ, but for aeroelastic work it is often said to be adequate to use the symmetric distribution in determining the damping in roll. The discovery that on a particular aircraft the use of the two distributions gave answers which differed by about 25 per cent. led to the consideration of more comprehensive methods for solving the general aeroelastic problem.

The method described herein is the culmination of this work. It is capable of being used either with lifting-surface or lifting-line theory. The exact method of adapting it to lifting-surface theory is not defined since it would require a definition

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of the distinction between the two theories—a problem which is considered more fully elsewhere[3]. In general a mathematical treatment is avoided where possible, but it has been considered essential to show the derivation of the basic integral equation. The method of solution is by means of the interpolation polynomials employed originally by Multhopp in his solution of the lifting-surface problem. For this it is necessary to define a continuous function of two variables known as the elastic influence function. The broad definition is included here but its derivation from a series of point-by-point values has not been included.

The integral equation is satisfied at fifteen spanwise stations only. The solution of this problem leads to seven or eight simultaneous equations whose general form is shown. The form for any other number of stations has not been sought as this number was quite adequate for the fighter aircraft under consideration. An application of the method has been included as a worked example and a comparison between the more conventional form of solution and the result of using this method on a swept-back wing has been shown graphically.

NOTATION AND ABBREVIATIONS

A	acceleration potential (in Section 4)
A	aspect ratio (in Sections 1, 2, 3)
a	speed of sound
b	span (ft.)
c.p.	centre of pressure
c	chord in line of flight (ft.)
c_0, c_t	root and tip chords, respectively
Θ, F, f	elastic influence function
x, y	spanwise co-ordinates
x_0, y_0	co-ordinates of doublet
x', y'	variables of integration
a_0	local lift derivative
$g_n(\theta)$	interpolation function used by Multhopp in Ref. 1
b_{vr}, b_{vn}	coefficients of Multhopp's equations in Ref. 1
C_L	local lift coefficient
$\overline{C}_L$	overall lift coefficient (symmetric loading)
$\overline{C}_l$	overall lift coefficient (anti-symmetric loading)
i, j	functions defined by equations (6) of the main paper
l	local load density
L	local shear load
p	pressure
$p(\eta'), p_1, p_n$	distance of c.p. from flexural axis (*see* Appendix)
p'	angular velocity in roll
M	Mach number
n, r, s, ν	integers which, as suffixes, refer to equations or variables
V	speed of aircraft
u, v, w	small perturbation velocities
w	downwash velocity at trailing edge
t	wing section thickness
t_{ns}	coefficients in aeroelastic equations
$R_{r(s)}$	the s^{th} residue in equation r
r_1	distance of doublet $l\,dx_0\,dy_0$ from point of measurement of A (*see* Section 4)
α_0	geometric incidence for rigid structure
$\Delta\alpha$	geometric distortion incidence
α_e	effective incidence
ε	angle of downwash
γ	non-dimensional air load $= C_L c/(2b)$
η', η	non-dimensional spanwise co-ordinate

Γ circulation
σ co-ordinate of inboard end of aileron (*see* Appendix)
ρ air density
θ spanwise parameter
δ aileron-wing angle
δ_0 rigid aileron-wing angle
μ non-dimensional pitching moment about quarter-chord line $=C_m c/(2b)$

1. DERIVATION OF THE BASIC EQUATION

For the derivation of the fundamental aeroelastic equations a brief recapitulation of the lifting-line theory will be given. This theory supposes that the air flow over the wing surfaces generates a vortex sheet which has bound vortices contained within the plan form of the wing and trailing vortices which are their continuations. The theory depends on three hypotheses which are:

(*i*) The bound vortices are parallel to the span

(*ii*) The trailing vortices leave the trailing edge in lines which are parallel to the direction of motion of the aerofoil.

(*iii*) The velocity at any point P of the wing surface induced by trailing vortices may be replaced by the velocity induced at the trailing edge of the profile section through P. This last is known as the lifting-line hypothesis since it reduces the wing surfaces to an equivalent line.

Now it is clear that the induced velocity δw at any point P of the trailing edge due to these vortices at another point Q will depend on the distance between P and Q. Consequently it can be shown that

$$w(y)=\frac{1}{4\pi}\int_{-b/2}^{b/2}\frac{d\Gamma(x)}{(y-x)},$$

where $d\Gamma(x)$ is the circulation due to the amount of vorticity shed into the wake by the bound vortices.

For a non-dimensional treatment it is convenient to define

$$\gamma=\frac{C_l c}{2b}=\frac{\Gamma}{bV}$$

and to replace the spanwise co-ordinate y by η where

$$\frac{b}{2}\eta=y,$$

b being the span and y being positive to starboard.

Consequently, since it may be assumed that the downwash velocity is small relative to the forward velocity of the aerofoil, the angle of downwash ε is given by

$$\varepsilon(\eta)=\frac{w}{V}=\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma(\eta')}{d\eta'}\frac{d\eta'}{(\eta-\eta')}$$

The effective incidence α_e is given by the difference between the geometric and the induced incidence.

i.e. $$\alpha_e=\alpha_0-\varepsilon.$$

Consequently $$\alpha_e=\alpha_0-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta-\eta')}.$$

Now by definition

$$\gamma = \frac{C_L c}{2b} = \alpha_e a_0 \frac{c}{2b} \qquad \text{where} \quad a_0 = \frac{\partial C_L}{\partial \alpha}.$$

Then

$$\frac{2b}{a_0 c}\gamma(\eta) = \alpha_0 - \frac{1}{2\pi}\int_{-1}^{+1} \frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta - \eta')} \quad . \quad . \quad . \quad (1)$$

This is the well-known integral equation for which an approximate solution by means of interpolation functions has been deduced by Multhopp. He derived the equation

$$\left[b_{\nu\nu} + \frac{2b}{a_0 c}\right]\gamma_\nu = \alpha_\nu + \sum_{n=1}^{m}{}' \, b_{\nu n}\gamma_n \quad . \quad . \quad . \quad (2)$$

where Σ' denotes $n \neq \nu$.

Consider now the introduction of an elastic influence function $f(\eta, \eta')$ which defines the incidence change at η due to a unit load on the c.p. locus at η'. If the load distribution is defined as positive upwards and is written $l(\eta')$, then the incidence change due to distortion at any point of one wing is given by

$$\Delta\alpha(\eta) = \frac{b}{2}\int_0^1 l(\eta') f(\eta, \eta')\, d\eta' \quad . \quad . \quad . \quad . \quad (3)$$

it being presumed that the loading on one wing in steady state has no influence upon the distortion of the other wing. This is not strictly true, since there frequently occurs a difference between symmetric and anti-symmetric wing torsional stiffnesses. If there were a marked difference it would be possible to define two functions $f_1(\eta, \eta')$ and $f_2(\eta, \eta')$ for symmetric and anti-symmetric loading respectively. However, this is a point of detail and is not essential to the main argument.

By definition

$$\frac{dL}{dy} = l(\eta') = \tfrac{1}{2}\rho V^2 C_L c = \rho V^2 b\gamma(\eta'),$$

so that

$$\Delta\alpha(\eta) = \tfrac{1}{2}\rho V^2 b^2 \int_0^1 \gamma(\eta') f(\eta, \eta')\, d\eta'.$$

Consequently, the local geometric incidence being given by $\alpha_0 + \Delta\alpha$, it follows that for $\eta \geqslant 0$,

$$\frac{2b}{a_0 c}\gamma(\eta) = \alpha_0 + \tfrac{1}{2}\rho V^2 b^2 \int_0^1 \gamma(\eta') f(\eta, \eta')\, d\eta' - \frac{1}{2\pi}\int_{-1}^{+1} \frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta - \eta')} \quad . \quad (4)$$

This equation will be known as the integral equation of the elastic aircraft. The significance of the different limits of the two integrals lies in the fact that the elastic influence function is only defined so long as η and η' lie on the same wing, but the angle of downwash on one is affected by the circulation on the other. It is, however, convenient to isolate the two cases of symmetric and anti-symmetric flight. This can always be done, and in these circumstances

(*i*) Symmetric loading implies $\gamma(\eta) = \gamma(-\eta)$

(*ii*) Anti-symmetric loading implies $\gamma(\eta) = -\gamma(-\eta)$

In this case it is possible to reduce the limits of integration in the last integral to those of the first.

2. METHOD OF SOLUTION

The method used by Multhopp in deriving the set of equations (2) from equations (1) depends upon an approximation to $\gamma(\eta)$ by means of continuous interpolation functions, which satisfies the integral equation (1) at a series of pivotal points. The particular distribution of these points along the span is a necessary adjunct to the form of the interpolation functions. An added feature using this method is that the subsequent simultaneous equations are of a form which can be solved by iterative processes.

The solution of equation (4) has been attempted using the same interpolation functions to give a continuous representation for the elastic influence function. The particular case of 15 spanwise pivotal points from tip to tip was chosen and these were distributed according to the law

$$\eta_r = \cos\theta_r \qquad \text{where} \quad \theta_r = \frac{r\pi}{16}$$

for $1 \leqslant r \leqslant 15$. It will be remembered that η is the non-dimensional co-ordinate given by

$$y = \frac{b}{2}\eta \qquad \text{so that if } -\frac{b}{2} \leqslant y \leqslant \frac{b}{2} \qquad \text{then } -1 \leqslant \eta \leqslant 1.$$

If $f(\eta_r, \eta'_s)$ is denoted by f_{rs} it defines the incidence change at η_r due to unit load on the c.p. at η'_s. It follows, by what has been said previously, since $\eta_8 = 0$ and since the elastic influence does not pass from one wing to the other, that f_{rs} has no meaning if $1 \leqslant r \leqslant 8$ and $8 \leqslant s \leqslant 15$. In particular, since the aircraft centre line is chosen as the axis of reference for incidence change, f_{8s} and f_{rs} are zero for all values of r and s.

It therefore follows that a complete representation of the elastic influence function can be deduced from f_{rs} where $1 \leqslant r \leqslant 8$, $1 \leqslant s \leqslant 8$ with zero values if either or both r and s are 8. Using the continuous representation based on the point-by-point values, a set of simultaneous equations has been deduced in a form analogous to equations (2). The coefficients calculated for these equations are independent of wing geometry or stiffness so that the form is generally applicable. The equations become

$$\left[b_{vv} + \frac{2b}{a_0 c}\right]\gamma_v = \alpha_v + \tfrac{1}{2}\rho V^2 b^2 \left\{ \sum_{n=1}^{8} \left[\gamma_n \sum_{s=1}^{8} f_{vs}\, t_{ns}\right]\right\} + \sum_{n=1}^{8}{}' b_{vn}\gamma_n \qquad . \qquad (5)$$

where t_{ns} are the "once for all" coefficients resulting from the continuous representation of $\gamma(\eta)$ and $f(\eta, \eta')$, and Σ' again denotes $n \neq v$. These equations are valid for $1 \leqslant v \leqslant 8$ and two sets have been deduced, one for the symmetric case and one for the anti-symmetric. The terms α_v, of course, are the values of the incidence at the various pivotal points when the wing is undistorted. The standard set of values of b_{vv} and b_{vn} deduced by Multhopp for the symmetric and anti-symmetric cases are set out in Tables I and II.

The values of the coefficients t_{ns} for the symmetric and anti-symmetric cases have been calculated and are given in Tables III.

Consideration of these tables will show that the convenient form of equations (2) which permits an iterative solution is unfortunately lost. However, it usually happens that some comparison with the rigid wing is required, in which case equations (2) will require solution. It is suggested that since the elastic influence must not predominate, the solution of equations (5) will not differ by much more than 30 per cent. from the rigid solution. Consequently solution of equations (5) can be achieved using relaxation methods with the initial estimates being the solution of equations (2). The form of the simultaneous equations is particularly suited to relaxation methods for there exists in each one a coefficient in which γ predominates over all the others so that it is decided unequivocally which influences most the value of each residue.

TABLE I

ν	$b_{\nu\nu}$ *Symmetric*	$b_{\nu\nu}$ *Anti-symmetric*
1	20.503	20.503
2	10.453	10.453
3	7.200	7.200
4	5.657	5.657
5	4.811	4.811
6	4.330	4.330
7	4.079	4.079
8	4.000	—

TABLE IIA

$b_{\nu n}$ SYMMETRIC

$n=$	1	2	3	4	5	6	7	8
$\nu=1$	0	7.392	0	0.606	0	0.193	0	0.065
2	3.769	0	4.077	0	0.407	0	0.164	0
3	0	2.809	0	2.876	0	0.326	0	0.090
4	0.167	0	2.260	0	2.296	0	0.309	0
5	0	0.187	0	1.952	0	1.998	0	0.203
6	0.041	0	0.196	0	1.798	0	1.925	0
7	0	0.064	0	0.223	0	1.814	0	1.642
8	0.025	0	0.101	0	0.337	0	3.221	0

TABLE IIB

$b_{\nu n}$ ANTI-SYMMETRIC ($\gamma_8=0$)

$n=$	1	2	3	4	5	6	7
$\nu=1$	0	7.379	0	0.575	0	0.130	0
2	3.762	0	4.055	0	0.359	0	0.067
3	0	2.793	0	2.839	0	0.248	0
4	0.159	0	2.231	0	2.231	0	0.159
5	0	0.165	0	1.897	0	1.866	0
6	0.028	0	0.149	0	1.680	0	1.558
7	0	0.026	0	0.114	0	1.468	0

TABLE IIIA

t_{ns} SYMMETRIC

$n=$	1	2	3	4	5	6	7	8
$s=1$	0.01702	0.00394	−0.00380	0.00372	−0.00372	0.00390	−0.00432	0.00308
2	0.00383	0.03514	0.00480	−0.00500	0.00525	−0.00576	0.00670	−0.00528
3	−0.00352	0.00457	0.05174	0.00561	−0.00609	0.00698	−0.00849	0.00735
4	0.00321	−0.00443	0.00521	0.06626	0.00653	−0.00785	0.01002	−0.00962
5	−0.00291	0.00419	−0.00507	0.00582	0.07813	0.00870	−0.01157	0.01254
6	0.00263	−0.00391	0.00487	−0.00580	0.00714	0.08699	0.01270	−0.01683
7	−0.00229	0.00347	−0.00436	0.00524	−0.00647	0.00832	0.09246	0.02603

TABLE IIIB

t_{ns} ANTI-SYMMETRIC

$n=$	1	2	3	4	5	6	7
$s=1$	0.01736	0.00324	−0.00271	0.00220	−0.00170	0.00123	−0.00071
2	0.00335	0.03614	0.00325	−0.00282	0.00230	−0.00179	0.00110
3	−0.00299	0.00347	0.05346	0.00315	−0.00269	0.00224	−0.00148
4	0.00271	−0.00339	0.00355	0.06869	0.00306	−0.00280	0.00200
5	−0.00252	0.00336	−0.00372	0.00377	0.08123	0.00382	−0.00292
6	0.00251	−0.00364	0.00435	−0.00485	0.00537	0.09050	0.00464
7	−0.00273	0.00433	−0.00562	0.00678	−0.00802	0.00902	0.09629

3. WORKED EXAMPLE

A complete example will now be worked out on an un-swept wing, the values used for the stiffness terms having no relation to any given aircraft. The symmetric lift of a given wing will be calculated. The wing geometry is as follows:—

Semi span	=	$b/2$	=	19.5 ft.
Aspect ratio	=	A	=	5.52
Tip chord	=	c_t	=	3.89 ft.
Root chord	=	c_0	=	10.246 ft.
General chord	=	10.246	−	$6.356\,\eta$ ft.

The value of a_0 will be taken as constant and equal to $2\pi(1+0.8\,t/c)$

Thickness ratio	=	t/c	=	10 per cent.
Then a_0	=	6.786.		

Table IV gives values necessary for the solution of the rigid problem, for 15 spanwise pivotal points.

The lift distribution will be considered for low speed so that Mach number contraction need not be employed. The rigid incidence will be regarded as unity so that the lift derivative will be directly equal to the lift coefficient deduced. The set of equations (2) becomes

Group I

$$\left.\begin{aligned}
&(i) && 23.368\gamma_1 = 1 + 7.392\gamma_2 + 0.606\gamma_4 + 0.193\gamma_6 + 0.065\gamma_8 \\
&(iii) && 9.517\gamma_3 = 1 + 2.809\gamma_2 + 2.876\gamma_4 + 0.326\gamma_6 + 0.090\gamma_8 \\
&(v) && 6.523\gamma_5 = 1 + 0.187\gamma_2 + 1.952\gamma_4 + 1.998\gamma_6 + 0.203\gamma_8 \\
&(vii) && 5.355\gamma_7 = 1 + 0.064\gamma_2 + 0.223\gamma_4 + 1.814\gamma_6 + 1.642\gamma_8
\end{aligned}\right\}$$

Group II

$$\left.\begin{aligned}
&(ii) && 13.081\gamma_2 = 1 + 3.769\gamma_1 + 4.077\gamma_3 + 0.407\gamma_5 + 0.164\gamma_7 \\
&(iv) && 7.655\gamma_4 = 1 + 0.167\gamma_1 + 2.260\gamma_3 + 2.296\gamma_5 + 0.309\gamma_7 \\
&(vi) && 5.801\gamma_6 = 1 + 0.041\gamma_1 + 0.196\gamma_3 + 1.798\gamma_5 + 1.925\gamma_7 \\
&(viii) && 5.122\gamma_8 = 1 + 0.025\gamma_1 + 0.101\gamma_3 + 0.337\gamma_5 + 3.221\gamma_7
\end{aligned}\right\}$$

The iterative solution of these equations is obtained by guessing values for γ_2, γ_4, γ_6, and γ_8 and substituting in Group I to obtain values of γ_1, γ_3, γ_5 and γ_7 which in turn are substituted in Group II. The process is continued until the result of substitutions makes no alteration of the values thus obtained.

To a three-figure accuracy the result of so solving these equations is

$$\begin{aligned}
\gamma_1 &= 0.134 & \gamma_2 &= 0.235 \\
\gamma_3 &= 0.316 & \gamma_4 &= 0.385 \\
\gamma_5 &= 0.452 & \gamma_6 &= 0.510 \\
\gamma_7 &= 0.559 & \gamma_8 &= 0.583.
\end{aligned}$$

It is now necessary to consider the elastic influence terms at the pivotal stations. The speed for which the problem will be solved is $M=0.5$ at ground level.

It is assumed that the quarter-chord line is parallel to, and forward of, the flexural axis of the wing. Had it been aft the signs of the influence terms would

TABLE IV

v	$\eta=\cos\frac{v\pi}{16}$	c	a_0c	$\frac{2b}{a_0c}$	$\frac{2b}{a_0c}+b_{vv}$
1	0.9809	4.012	27.225	2.865	23.368
2	0.9239	4.374	29.682	2.628	13.081
3	0.8315	4.961	33.665	2.317	9.517
4	0.7071	5.752	39.033	1.998	7.655
5	0.5556	6.715	45.568	1.712	6.523
6	0.3827	7.814	53.026	1.471	5.801
7	0.1951	9.006	61.115	1.276	5.355
8	0	10.246	69.529	1.122	5.122

TABLE V

VALUES OF $f_{vs} \times 10^6$

$v=$	1	2	3	4	5	6	7
$s=1$	20	17	12	8	5	3	1
2	17	17	12	8	5	3	1
3	12	12	12	8	5	3	1
4	8	8	8	8	5	3	1
5	5	5	5	5	5	3	1
6	3	3	3	3	3	3	1
7	1	1	1	1	1	1	1

have been reversed. Since the wing has no sweepback, only torsion needs to be considered for incidence change. This gives a symmetry to the matrix of influence terms (Table V)

In order to illustrate the formation of the coefficients of the γ's in the elastic part of equations (5), a few are worked out in detail. The terms in question are

$$\tfrac{1}{2}\rho V^2 b^2 \sum_{n=1}^{8} \gamma_n \left(\sum_{s=1}^{8} f_{vs} t_{ns} \right)$$

where v denotes the particular equation being considered. Since the aircraft is to be considered flying at a Mach number of 0.5 at ground level,

$$\tfrac{1}{2}\rho a^2 M^2 = 0.375 \times 10^3.$$

Then

$$\tfrac{1}{2}\rho V^2 b^2 = 0.570 \times 10^6.$$

In the first equation $v=1$ so that the coefficient of $\tfrac{1}{2}\rho V^2 b^2 \gamma_3$ in this equation is

$$\begin{aligned} &-20 \times 0.00380 \times 10^{-6} \\ &+17 \times 0.00480 \times 10^{-6} \\ &+12 \times 0.05174 \times 10^{-6} \\ &+8 \times 0.00521 \times 10^{-6} \\ &-5 \times 0.00507 \times 10^{-6} \\ &+3 \times 0.00487 \times 10^{-6} \\ &-1 \times 0.00436 \times 10^{-6} \\ \hline &= 0.65306 \times 10^{-6} \end{aligned}$$

and hence the coefficient of $\gamma_3 = 0.570 \times 0.65306$
$= 0.372.$

In the same equation the coefficient of $\tfrac{1}{2}\rho V^2 b^2 \gamma_8$ is

$$\begin{aligned} &20 \times 0.00308 \times 10^{-6} \\ &-17 \times 0.00528 \times 10^{-6} \\ &+12 \times 0.00735 \times 10^{-6} \\ &-8 \times 0.00962 \times 10^{-6} \\ &+5 \times 0.01254 \times 10^{-6} \\ &-3 \times 0.01683 \times 10^{-6} \\ &+1 \times 0.02603 \times 10^{-6} \\ \hline &= 0.02132 \times 10^{-6} \end{aligned}$$

and hence the coefficient of $\gamma_8 = 0.570 \times 0.02132$
$= 0.012.$

Finally, the coefficient of $\frac{1}{2}\rho V^2 b^2 \gamma_5$ in equation (6) is

$$\begin{aligned}
&-3\times 0.00372\times 10^{-6}\\
&+3\times 0.00525\times 10^{-6}\\
&-3\times 0.00609\times 10^{-6}\\
&+3\times 0.00653\times 10^{-6}\\
&+3\times 0.07813+10^{-6}\\
&+3\times 0.00714\times 10^{-6}\\
&-1\times 0.00647\times 10^{-6}\\
&\quad =0.25525\times 10^{-6}
\end{aligned}$$

and hence the coefficient of $\gamma_5 = 0.570 \times 0.25525$

$$= 0.145 .$$

The full matrix of coefficients for γ_n in equation ν due to the elastic terms is given by Table VI.

Adding these coefficients to those of the corresponding γ's on the right hand side of the equations previously determined for the rigid wing, the elastic equations are

$$\begin{aligned}
&(i) && 0=1-23.151\gamma_1+7.796\gamma_2+0.372\gamma_3+0.950\gamma_4+0.228\gamma_5+0.372\gamma_6+0.045\gamma_7+0.077\gamma_8\\
&(ii) && 0=1+3.956\gamma_1-12.684\gamma_2+4.456\gamma_3+0.338\gamma_4+0.641\gamma_5+0.172\gamma_6+0.216\gamma_7+0.007\gamma_8\\
&(iii) && 0=1+0.128\gamma_1+3.095\gamma_2-9.141\gamma_3+3.217\gamma_4+0.230\gamma_5+0.504\gamma_6+0.045\gamma_7+0.103\gamma_8\\
&(iv) && 0=1+0.256\gamma_1+0.186\gamma_2+2.516\gamma_3-7.323\gamma_4+2.536\gamma_5+0.166\gamma_6+0.368\gamma_7+0.001\gamma_8\\
&(v) && 0=1+0.053\gamma_1+0.306\gamma_2+0.157\gamma_3+2.163\gamma_4-6.286\gamma_5+2.169\gamma_6+0.053\gamma_7+0.212\gamma_8\\
&(vi) && 0=1+0.074\gamma_1+0.066\gamma_2+0.292\gamma_3+0.124\gamma_4+1.943\gamma_5-5.647\gamma_6+1.986\gamma_7\quad 0\\
&(vii) && 0=1+0.010\gamma_1+0.088\gamma_2+0.030\gamma_3+0.266\gamma_4+0.046\gamma_5+1.872\gamma_6-5.299\gamma_7+1.652\gamma_8\\
&(viii) && 0=1+0.025\gamma_1+0.101\gamma_3+0.337\gamma_5+3.221\gamma_7-5.122\gamma_8
\end{aligned}$$

Suppose now the values determined for the rigid case be substituted in the expressions on the right hand side. In general the result will differ from zero and this difference is called the residue. It will be noticed that the biggest coefficient in equation r is that of γ_r. Consequently if the residue of each expression be divided by the biggest coefficient, an approximate alteration of the corresponding γ's will be found necessary to reduce the residues. Repeating this process with the fresh residues the values of the γ's necessary to satisfy these equations to any required accuracy can be found. The first steps in this process are shown in Table VII.

The columns in Table VII are constructed on the following basis. Column 1 denotes the number of the equation and the γ to be modified. Column 2 shows the first approximation to the solution of the equations. The values used are in fact those previously derived in the rigid case. Column 3 shows the residue in each expression on substituting the first approximation. Column 4 shows the quotients on dividing each residue by the biggest coefficient in the corresponding expression. Column 5 shows the resulting residues on substituting the sums of corresponding terms in Columns 2 and 4 in the expressions. This process is carried on until a sufficiently

TABLE VI

COEFFICIENTS FOR γ_n

$n=$	1	2	3	4	5	6	7	8
$\nu=1$	0.217	0.404	0.372	0.344	0.228	0.179	0.045	0.012
2	0.187	0.397	0.379	0.338	0.234	0.172	0.052	0.007
3	0.128	0.286	0.376	0.341	0.230	0.178	0.045	0.013
4	0.089	0.186	0.256	0.332	0.240	0.166	0.059	0.001
5	0.053	0.119	0.157	0.211	0.237	0.171	0.053	0.009
6	0.033	0.066	0.096	0.124	0.145	0.154	0.061	0
7	0.010	0.024	0.030	0.043	0.046	0.058	0.056	0.010
8	0	0	0	0	0	0	0	0

TABLE VII

r	$\gamma_{r(0)}$	$R_{r(0)}$	$d\gamma_{r(1)}$	$R_{r(1)}$	$d\gamma_{r(2)}$	*Solution*
1	0.134	0.5760	0.025	0.4743	0.020	0.232
2	0.235	0.5899	0.046	0.4423	0.035	0.336
3	0.316	0.5407	0.059	0.4019	0.044	0.508
4	0.385	0.4909	0.067	0.3317	0.045	0.611
5	0.452	0.3795	0.060	0.2793	0.044	0.671
6	0.510	0.2739	0.049	0.1859	0.033	0.699
7	0.559	0.1104	0.021	0.1175	0.022	0.696
8	0.583	0.0020	0	0.0964	0.019	0.688

accurate set of values for the γ's is obtained. Although this rule does finally give the required answer, the rate of convergence is slow and it would be convenient to devise a rule more suited to the particular equations. The purpose in showing this rule is that it is applicable to all the sets of equations that occur in this type of problem. Any other rule would not have this generality.

Having obtained the solutions for the γ's it is but a short step to obtain the lift coefficient for the wing.

Knowing that $$\gamma = \frac{C_L c}{2b},$$

it follows that $$\bar{C}_L = A \int_{-1}^{+1} \gamma d\eta,$$

where $\bar{C}_L$ is the overall lift coefficient and A is the aspect ratio. This formula applies only in the symmetric loading case, since anti-symmetric conditions imply

$$\int_{-1}^{0} \gamma d\eta = -\int_{0}^{1} \gamma d\eta,$$

in which case $\bar{C}_l$ is defined as the coefficient of roll and is

$$\bar{C}_l = \frac{A}{2} \int_{-1}^{1} \gamma \eta d\eta.$$

Owing to the use of interpolation functions the integration reduces to

$$\bar{C}_L = \frac{2\pi A}{16} \left\{ \frac{\gamma_8}{2} + \sum_{1}^{7} \gamma_n \cos\left(\frac{\pi}{2} - \frac{n\pi}{16}\right) \right\}$$

and $$\bar{C}_l = \frac{\pi A}{32} \left\{ \sum_{1}^{7} \gamma_n \sin\left(\pi - \frac{n\pi}{8}\right) \right\},$$

for this particular case of 15 spanwise stations. If, for example, the rigid incidence is taken as unity

$$\bar{C}_L = \frac{\partial \bar{C}_L}{\partial \alpha},$$

or if the aileron angle δ is chosen as unity

$$\bar{C}_l = \frac{\partial \bar{C}_l}{\partial \delta}.$$

For the rigid wing

$$\bar{C}_L = \frac{\pi \times 5.52}{8} [0.5 \times 0.583 + 0.9808 \times 0.559 + 0.9239 \times 0.510 + 0.8315 \times 0.452 + \\ + 0.7071 \times 0.385 + 0.5556 \times 0.316 + 0.3827 \times 0.235 + 0.1951 \times 0.134] \\ = 4.879$$

For the elastic wing

$$\bar{C}_L = \frac{\pi \times 5.52}{8}[0.5 \times 0.688 + 0.9808 \times 0.696 + 0.9239 \times 0.699 + 0.8315 \times 0.671 + + 0.7071 \times 0.611 + 0.5556 \times 0.508 + 0.3827 \times 0.336 + 0.1951 \times 0.232] = 6.758 .$$

4. OUTLINE OF LIFTING SURFACE APPLICATION

It is perhaps desirable at this stage to discuss the application of the preceding results to Multhopp's method of Ref. 1. In brief Multhopp's process can be outlined as follows.

Given the basic aerodynamic equations in vectorial form,

$$\frac{d}{dt}(\mathbf{V}) + \frac{1}{\rho}\nabla p = 0$$

$$\frac{1}{\rho}\frac{d}{dt}(\rho) + \nabla . \mathbf{V} = 0 ,$$

where p = pressure field

ρ = density

$\mathbf{V} = (V + u, v, w)$, where u, v, w are small perturbation velocities.

In steady flow these reduce to

$$V\frac{\partial}{\partial x}(\mathbf{V}) + \frac{1}{\rho}\nabla p = 0$$

and

$$\frac{V}{\rho}\frac{\partial}{\partial x}(\rho) + \nabla . \mathbf{V} = 0 .$$

The acceleration potential is defined by

$$dA = \frac{dp}{\rho} ,$$

so that

$$\nabla A = \frac{1}{\rho}\nabla p .$$

Then

$$V\frac{\partial}{\partial x}(\mathbf{V}) = -\nabla A .$$

Now

$$\frac{dp}{d\rho} = a^2 ,$$

where a is the speed of sound, so that

$$\frac{V}{\rho}\frac{\partial \rho}{\partial x} = \frac{V}{\rho a^2}\frac{\partial p}{\partial x} = \frac{V}{a^2}\frac{\partial A}{\partial x} .$$

It follows that

$$\nabla . V\frac{\partial}{\partial x}(\mathbf{V}) = -\nabla^2 A$$

and

$$\frac{V}{a^2}\frac{\partial^2 A}{\partial x^2} = -\nabla . \frac{\partial}{\partial x}(\mathbf{V}) .$$

Whence

$$\nabla^2 A = \frac{V^2}{a^2}\frac{\partial^2 A}{\partial x^2} ,$$

or

$$(1 - M^2)\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} = 0 .$$

Clearly, if the Glauert contraction, previously referred to, is carried out by writing

$$x_1 = x$$
$$y_1 = \sqrt{(1-M^2)}\, y$$
$$z_1 = \sqrt{(1-M^2)}\, z,$$

this last equation reduces to

$$\frac{\partial^2 A}{\partial x_1^{\,2}} + \frac{\partial^2 A}{\partial y_1^{\,2}} + \frac{\partial^2 A}{\partial z_1^{\,2}} = 0, \quad M < 1.$$

So that it is only necessary to consider in general the solution of this form of equation, with the boundary condition

$$\frac{w}{V} = -\alpha,$$

the local incidence.
If the local non-dimensional load on the wing is defined by

$$l(x,y) = \frac{dp}{\frac{1}{2}\rho V^2},$$

then

$$\tfrac{1}{2}V^2 l(x,y) = dA.$$

This defines the discontinuity in the acceleration potential field across the wing. Referring to Green's theorem it follows that a solution for A is given by

$$A(x,y,z) = \frac{V^2}{8\pi}\int\int l(x_0,y_0)\frac{\partial}{\partial z}\left(\frac{1}{r_1}\right)dx_0 dy_0$$

where the double integral is taken over the wing and r_1 is the distance between the point at which A is being measured and the doublet of strength $l\,dx_0 dy_0$ at the point $(x_0, y_0, 0)$. The values of A over the wing where z tends to zero are considered. Furthermore, referring to the initial equations, it will be seen that

$$V\frac{\partial}{\partial x}(w) = -\frac{\partial A}{\partial z}.$$

Consequently

$$\frac{w}{V}(x,y,z) = -\frac{1}{V^2}\int_{-\infty}^{x}\frac{\partial A}{\partial z}(x',y,z)\,dx'.$$

Thus the final form of the integral equation for the wing loading becomes

$$\alpha = \frac{1}{V^2}\int_{-\infty}^{x}\frac{\partial A}{\partial z}(x',y,0)\,dx',$$

where

$$A(x,y,z) = \frac{V^2}{8\pi}\int\int l(x_0,y_0)\frac{\partial}{\partial z}\left(\frac{1}{r_1}\right)dx_0 dy_0.$$

But

$$\frac{\partial A}{\partial z}(x',y,0) = -\frac{V^2}{8\pi}\int\int l(x_0,y_0)\frac{1}{r_1^{\,3}}\,dx_0 dy_0,$$

where

$$r_1^{\,2} = (x'-x_0)^2 + (y-y_0)^2$$

and

$$\int_{-\infty}^{x}\frac{dx'}{r_1^{\,3}} = \frac{1}{(y-y_0)^2}\left[1+\frac{x-x_0}{r_1}\right].$$

Thus finally

$$\alpha = -\frac{1}{8\pi}\int\int\frac{l(x_0,y_0)}{(y-y_0)^2}\left[1+\frac{x-x_0}{r_1}\right]dx_0 dy_0.$$

Multhopp proceeds from here by defining two chordwise distributions $L_1(x_0, y_0)$ and $L_2(x_0, y_0)$ such that L_1 produces lift but no moment about the quarter-chord line and L_2 produces a moment but no lift. He then writes

$$\left.\begin{aligned} i(x,y,x_0,y_0) &= \frac{1}{C_L c}\int_{x_{0l}}^{x_{0t}} L_1(x_0,y_0)\left[1+\frac{x-x_0}{r}\right]dx_0 \\ j(x,y,x_0,y_0) &= \frac{1}{C_m c}\int_{x_{0l}}^{x_{0t}} L_2(x_0,y_0)\left[1+\frac{x-x_0}{r}\right]dx_0 , \end{aligned}\right\} \quad . \quad . \quad (6)$$

where x_{0l} and x_{0t} are the x co-ordinates of the leading and trailing edges corresponding to the point (x_0, y_0). The important point to realise is that the integral equation can now be expressed in the form

$$\alpha = -\frac{1}{8\pi}\left\{\int_{-b/2}^{b/2}\frac{C_L c(y')\, i(y,y')}{(y-y')^2}dy' + \int_{-b/2}^{b/2}\frac{C_m c(y')\, j(y,y')}{(y-y')^2}dy'\right\} .$$

In accord with the earlier parts of this paper, Multhopp uses

$$\mu = \frac{C_L c}{2b} , \qquad \mu = \frac{C_m c}{2b} .$$

Again using the interpolation functions which he denotes by $g_n(\theta)$, where it will be remembered that

$$\eta_r = \cos\theta_r \quad \text{and} \quad \theta_r = \frac{r\pi}{16} ,$$

the (γi) and (μj) distributions are defined by

$$(\gamma i) = \sum_{n=1}^{15} (\gamma i)_n\, g_n(\theta)$$

$$(\mu j) = \sum_{n=1}^{15} (\mu j)_n\, g_n(\theta) ,$$

a device which enables integration to be carried out before a precise knowledge of γ and μ is available.

Note in particular that $(\gamma i)_n = \gamma_n i_n$

where γ_n are not known at this stage and i_{vn} are derived from graphs at the end of Ref. 1. Similarly for $(\mu j)_n$. It should be pointed out that the numbering of the subscripts differs in this paper from that used by Multhopp. This is unfortunately unavoidable since it is used here to agree with Küchemann's report. The precise difference in the numbering of the pivotal points is as follows:—

Küchemann: 8 at the centre line, 1 at starboard tip, 15 at the port tip.

Multhopp: 0 at the centre line, 7, -7 at starboard and port tips respectively.

Returning to the method, after compensating for the logarithmic singularities, the equations derived by Multhopp are

$$\alpha_v = b_{vv}(\gamma_v \bar{i}_{vv} + \mu_v \bar{j}_{vv}) - \sum_{n=1}^{15}{}' b_{vn}(\gamma_n i_{vn} + \mu_n j_{vn}) .$$

The aeroelastic modification with which this paper is concerned is finally

$$\alpha_\nu + \tfrac{1}{2}\rho V^2 b^2 \{ \sum_{n=1}^{8} (\gamma_n \sum_{s=1}^{8} f_{\nu s} t_{ns} + c_n \mu_n \sum_{s=1}^{8} F_{\nu s} t_{ns}) \}$$
$$= b_{\nu\nu}(\gamma_\nu \bar{i}_{\nu\nu} + \mu_\nu \bar{j}_{\nu\nu}) - \sum_{n=1}^{8}{}' b_{\nu n}(\gamma_{\nu n} i_{\nu n} + \mu_n j_{\nu n}),$$

where the symbols $b_{\nu n}$, t_{ns} have the same values as previously, having one set of values for symmetric loading and another for anti-symmetric. The terms c_n are the local chords at stations n. The influence functions $f_{\nu s}$ and $F_{\nu s}$ are now defined more precisely as

$f_{\nu s}$ = the incidence change at ν due to unit load on the quarter-chord line at s.

$F_{\nu s}$ = the incidence change at ν due to unit couple about the quarter-chord line at s.

This device avoids the imprecise use of a c.p. locus.

This substitution follows since

$$\frac{dM}{dy} = \tfrac{1}{2}\rho V^2 C_m c^2 = \rho V^2 c b \mu,$$

(where M is here the pitching moment)

so that
$$(\Delta\alpha)_m = \tfrac{1}{2}\rho V^2 b^2 \int_0^1 \mu(\eta')\, c(\eta')\, F(\eta, \eta')\, d\eta',$$

where $(\Delta\alpha)_m$ is the incidence change due to pitching moments along the wing.

Referring to Ref. 1 it will be seen that two sets of equations have to be deduced, one for one spanwise set of pivotal points, the other for the second row. This means that 14 or 16 simultaneous equations have to be solved with this number of spanwise stations—a formidable task. The number of points could be reduced to 11 giving 10 or 12 simultaneous equations in all, corresponding to anti-symmetric or symmetric problems. This requires the evaluation of a separate table of values of t_{ns}. The reduction would cause a loss of accuracy in the result, but it is not likely, in the region where aeroelastic deformation is most noticeable, *i.e.* at high Mach numbers, that the inaccuracy would be great.

It is for this reason that Küchemann's method is preferred for swept wings, being effectively a lifting-line procedure based on experimental evidence. In fact the procedure of Ref. 2 is almost identical with that described at the beginning of this paper and it is felt unnecessary to explain its adaption to the aeroelastic problem.

CONCLUSION

Provided that the aerodynamic theory is adequate, the extension of this method to more and more spanwise stations will ultimately provide an adequate solution to every problem so tackled.

The finer points of aerodynamic theory have not been discussed. Such things as the effect of movements of the maximum camber line upon the position of the local centres of pressure have been deliberately ignored. It was felt that such refinements would only complicate the issue which, for this paper, is the consideration of the influence of structural elasticity in the light of a current aerodynamic theory. The test of the method will ultimately depend upon experiment, but in its absence the conclusions must be based upon the different solutions produced by this and by an older technique.

This method is capable of dealing with the symmetric and anti-symmetric lift distribution on swept and unswept wings, including the problem of aileron reversal. It is also suitable for assessing the behaviour of swept and unswept tailplanes on flexible fuselages and for determining the elevator reversal speed, if subsonic.

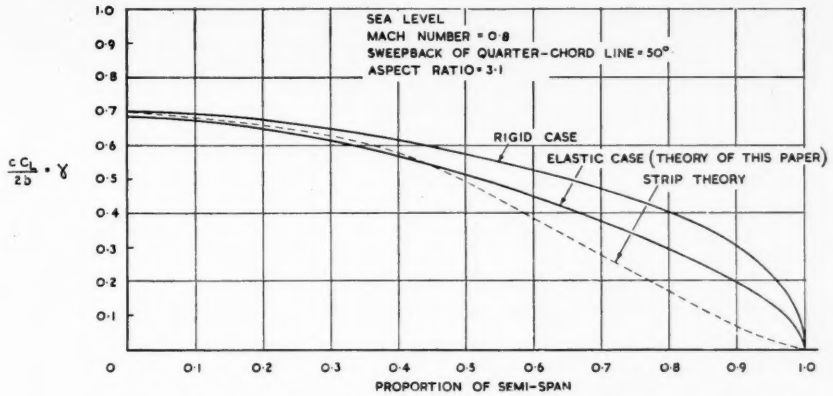

Fig. 1. Comparison of load distributions on the rigid wing and elastic wing computed by two methods.

However the existing lifting-surface theory seems to be rather inadequate when dealing with part-span ailerons, for the discontinuity of geometric incidence in the spanwise sense cannot be adequately described when using interpolation functions to represent the continuous γ distribution. This discontinuity may lead to end-plate effects which do already occur at the tips. (By end-plate effects is meant the modification of lift at the end of an aerofoil due to spanwise circulation). However, in the absence of more detailed experimental information a reasonable approximation may be achieved if the geometric incidence α due to aileron operation is factored to give an answer for the overall lift derivative which compares favourably with the experimental value. The pitching moment may likewise be adjusted by a suitable choice of the centre of pressure for the aileron load. Compressibility corrections can be applied up to the critical Mach number of the surface under consideration by a Glauert contraction in the directions perpendicular to the flight path.

The method has been applied to the problem of determining the symmetric lift distributions at a Mach number of 0.8 for a wing whose quarter-chord line is swept back at an angle of 50 degrees. Fig. 1 shows first the lift due to the rigid structure and then that computed for the elastic wing using this method. The dotted curve shows the distribution calculated using the rigid lift distribution in the conventional method. In relation to the method suggested here, the old treatment tends to over-emphasise the lift loss at the tip while neglecting completely the loss at the root consequent upon wing distortion. Owing to the relatively rapid movements of local centres of pressure near the root, this may have some important effects in computing the position of the aircraft aerodynamic centre for stability calculations.

While this paper has not been to any extent comprehensive it is to be hoped that the primary intention of illustrating the method has been achieved. Finally, while it is restricted to a particular distribution of pivotal points it is hoped that the adaptation of this method to experimental results has been indicated.

APPENDIX

It will be appreciated that the example given in detail is perhaps the simplest type which may occur. In order to illustrate the more complex side of the subject without over-burdening the text, a few of the points which have to be considered in the problem of rolling performance have been left to this Appendix.

To treat this problem with full rigour is beyond the scope of this paper for it will have to be assumed that sideslip effects are negligible and that the aircraft can

attain a steady rolling velocity—a state of affairs which cannot be obtained in practice. The alternative to these assumptions would be a knowledge of the oblique flow across the fuselage and the solution of an unsteady flow problem. It may be argued that the solution of this unrealistic problem is valueless, but it should be remembered that where aileron reversal is being considered the possibility of its occurrence within the transonic region can be discovered by the extrapolation of the graph of rolling velocity against Mach number obtained by such a method.

Returning to the mathematical treatment, it will be remembered that the integral equation

$$\alpha_e = (\alpha_0 + \Delta\alpha) - \varepsilon$$

was considered.

If δ denotes the angle between the aileron and the main surface, positive with trailing edge down, and the aileron extends from $\eta = \sigma$ to $\eta = 1$, then where

$$\frac{d\alpha}{d\delta} = \frac{\partial C_L}{\partial \delta} \Big/ \frac{\partial C_L}{\partial \alpha},$$

the geometric rigid incidence is given by

$$\alpha_0 = \frac{d\alpha}{d\delta}\delta_0, \quad \text{for} \quad \sigma \leqslant \eta \leqslant 1.$$

However, with an elastic structure the main surface will have a distortion $\Delta\alpha$ and the aileron similarly. Owing to the difficulty of estimating the hinge moments for such a structure it will further be assumed that the aileron distortion is negligible. A principle of superposition will be assumed in which the damping in roll and rolling moment due to aileron operation may be considered separately.

It follows that the geometric incidence of the structure is given by

$$\alpha = \Delta\alpha, \quad \text{for} \quad 0 \leqslant \eta \leqslant \sigma$$

$$\alpha = \frac{d\alpha}{d\delta}(\delta_0 - \Delta\alpha) + \Delta\alpha, \quad \text{for} \quad \sigma \leqslant \eta \leqslant 1.$$

Consider the limiting cases of no aileron, when $d\alpha/d\delta = 0$, and the case of all aileron, when $d\alpha/d\delta = 1$. In the first case,

$$\alpha = \Delta\alpha \quad \text{throughout}.$$

In the second case, $\alpha = \delta_0$, for $\sigma \leqslant \eta \leqslant 1$.

It follows that a solution of the integral equation

$$\alpha_e = \frac{d\alpha}{d\delta}(\delta_0 - \Delta\alpha) + \Delta\alpha - \varepsilon$$

must be sought. Thus it will depend upon a knowledge of

$$\Delta\alpha = \tfrac{1}{2}\rho V^2 b^2 \int_0^1 \gamma f(\eta, \eta')\, d\eta'.$$

Now $f(\eta, \eta')$ was defined as the incidence change at η due to unit load "on the centre of pressure" at η'. Hitherto a means of determining f has not been considered, but those familiar with the concept of a flexural axis can see that a suitable definition would be

$$f(\eta, \eta') = F(\eta, \eta') + p(\eta')\,\Theta(\eta, \eta')$$

where $F(\eta, \eta')$ defines the incidence change at η due to unit load on the flexural axis at η' and $\Theta(\eta, \eta')$ defines the incidence change at η due to unit couple at η'

about the flexural axis. Clearly the $p(\eta')$ defines the distance between the centre of pressure and the flexural axis at η' with a suitable sign convention.

The mainplane load itself will be on a given c.p. locus while the aileron induced load will be on another. The total load, being the sum of these two, will be applied at a centre of pressure locus which will depend upon the proportion of each load in the total. Suppose γ_1 denotes the mainplane load at a c.p. locus p_1 and γ_2 denotes the aileron load at a c.p. locus p_2. Then the local c.p. will be equal to

$$\frac{p_1\gamma_1+p_2\gamma_2}{\gamma} \quad \text{since} \quad \gamma_1+\gamma_2=\gamma.$$

It then follows that

$$\Delta\alpha=\tfrac{1}{2}\rho V^2b^2\int_0^1 \gamma f(\eta,\eta')\,d\eta'$$

$$=\tfrac{1}{2}\rho V^2b^2\int_0^1 [\gamma F+(\gamma_1p_1+\gamma_2p_2)\Theta]\,d\eta' \quad . \quad . \quad . \quad (1)$$

Let it be assumed that γ_1 is the solution of

$$\frac{2b}{a_0c}\gamma=\Delta\alpha-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta-\eta')} \quad . \quad . \quad . \quad . \quad (2)$$

and γ_2 is the solution of

$$\frac{2b}{a_0c}\gamma=(\delta_0-\Delta\alpha)\frac{d\alpha}{d\delta}-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta-\eta')} \quad . \quad . \quad . \quad (3)$$

so that the sum of (2) and (3) gives the full equation

$$\frac{2b}{a_0c}\gamma=\frac{d\alpha}{d\delta}(\delta_0-\Delta\alpha)+\Delta\alpha-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma}{d\eta'}\frac{d\eta'}{(\eta-\eta')}.$$

Substituting from (1) in (2) and (3), two equations are derived:—

$$\frac{2b}{a_0c}\gamma_1=\tfrac{1}{2}\rho V^2b^2\int_0^1[(\gamma_1+\gamma_2)F+(\gamma_1p_1+\gamma_2p_2)\Theta]d\eta'-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma_1}{d\eta'}\frac{d\eta'}{(\eta-\eta')} \quad . \quad (4)$$

and

$$\frac{2b}{a_0c}\gamma_2=\frac{d\alpha}{d\delta}\left\{\delta_0-\tfrac{1}{2}\rho V^2b^2\int_0^1[(\gamma_1+\gamma_2)F+(\gamma_1p_1+\gamma_2p_2)\Theta]\,d\eta'\right\}-\frac{1}{2\pi}\int_{-1}^{+1}\frac{d\gamma_2}{d\eta'}\frac{d\eta'}{(\eta-\eta')}$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It follows that two influence functions may now be defined.

$$f_{(1)}=F+p_1\Theta$$

and

$$f_{(2)}=F+p_2\Theta,$$

in which case (4) and (5) reduce to

$$\frac{2b}{a_0c}\gamma_1=\tfrac{1}{2}\rho V^2b^2\int_0^1[\gamma_1f_{(1)}+\gamma_2f_{(2)}]\,d\eta'-\frac{1}{2\pi}\int_{-1}^{1}\frac{d\gamma_1}{d\eta'}\frac{d\eta'}{(\eta-\eta')} \quad . \quad . \quad (6)$$

$$\frac{2b}{a_0c}\gamma_2=\frac{d\alpha}{d\delta}\left\{\delta_0-\tfrac{1}{2}\rho V^2b^2\int_0^1[\gamma_1f_{(1)}+\gamma_2f_{(2)}]\,d\eta'\right\}-\frac{1}{2\pi}\int_{-1}^{1}\frac{d\gamma_2}{d\eta'}\frac{d\eta'}{(\eta-\eta')} \quad . \quad (7)$$

Using the rule of construction of the simultaneous equations described in the main paper, two sets of simultaneous equations in γ_1 and γ_2 can be derived whose solution provides an approximation to the correct load distribution. However it may happen that $\Delta\alpha$ may justifiably be neglected in relation to δ_0 in the term $(\delta_0 - \Delta\alpha)\,d\alpha/d\delta$. In such circumstances it would be possible to solve (7) for γ_2 neglecting the aero-elastic integral, and to substitute the results in (6) to obtain a solution for γ_1.

This is not the whole story since the damping in roll has to be considered. This is derived from the anti-symmetric set of equations using $[p'b/(2V)]\,\eta$ as the local undistorted incidence, where p' is the angular velocity in roll measured positively when the starboard tip moves up. Calling γ^+ the load distribution for unit value of $p'b/(2V)$ it remains to solve

$$\int_0^1 \eta\,(\gamma_1 + \gamma_2 - \frac{pb}{2V}\gamma^+)\,d\eta = 0$$

a simple equation for $\dfrac{p'b}{2V}$.

Acknowledgment

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*A Note on the Vortex Patterns in the Boundary Layer Flow of a Swept-back Wing**

by

JOSEPH BLACK

1. INTRODUCTION

Detailed investigations of the two-dimensional characteristics of thin aerofoil sections with small leading-edge radius have been made by Gault and McCullough[1,2]. They found that at angles of incidence well below the stalling angle of the section, the laminar boundary layer separated from the surface near the nose, became transitional away from the surface, and re-attached to the aerofoil as a turbulent layer a little farther downstream. Beneath the separated layer, a "bubble" vortex was formed lying parallel to the leading edge, its rotation being such that the flow adjacent to the surface was towards the leading edge. This vortex extended only over a very narrow chordwise region, and with increase of incidence, it moved towards the leading edge, becoming narrower. The stalling in the case of one section appeared to arise with the failure of the separated layer to re-attach, even after it had become turbulent, and was not due to any turbulent separation farther aft.

Pressure measurements on swept-back wings using similar sections have been reported, and have indicated that laminar separation bubble vortices were being formed, but since the vortices are small and localised, wool tufts have not satisfactorily revealed their exact location, nor the behaviour of the flow within the vortex.

In the course of a wind-tunnel investigation of a tapered wing with 44° sweepback of the leading edge (aerofoil section 10 per cent. thick at 40 per cent. chord) at a Reynolds number of 0.5×10^6, the author was able to study the growth of the vortices in the boundary layer, and the flow within them, by use of a well-known technique. This was to spray a suspension of lamp-black in paraffin onto the wing surface and then to turn the wind on rapidly. The flow in the boundary layer was clearly revealed by the movement of the liquid, and after a few minutes, the paraffin evaporated, leaving a pattern of dried lamp-black which could be studied at leisure with the wind off.

Most unexpected flow conditions, especially at the higher incidences, were discovered in this way, and as they are believed to be unknown, and hitherto unsuspected, it was considered desirable to report their existence as early as possible in this preliminary note. It is hoped to report the complete investigation at a later date.

2. THE BOUNDARY LAYER FLOW

Sketches and photographs illustrating the development of the patterns are shown in Figs. 1 and 2.

The first evidence of the formation of a bubble vortex appeared at 6° incidence. The wing leading edge was scrubbed clean back to about 0.02 chord, and immediately behind the scrubbed area, a narrow band of lamp-black accumulated over a spanwise section of the inboard leading edge, and farther outboard from about 0.8 semi-span to the tip. Within the vortex, there was a strong spanwise flow which pulled the liquid from the wing apex, even with the wing mounted vertically. The vortex followed the curve of the wing tip back to the trailing edge. The

*Unexpected behaviour of the boundary layer flow pattern was found by the author during a recent experimental investigation on a swept-back wing. The photographs indicate that this pattern is different from that which has been generally accepted in the past few years. The work is to be written up at length, but it was considered important that this information should be made known quickly.

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spanwise regions between the two vortices appeared to have normal laminar-turbulent transition on the surface.

At 8° the vortex extended the whole length of the leading edge, except near the root, where it curved aft before fading out. The band of lamp-black tapered, with its broadest width inboard, and approached the leading edge towards the tip. While the wind was on, and before the liquid dried, a thin column of liquid could be clearly seen flowing steadily right along the leading edge and accumulating in a small patch at the leading edge tip. This accumulated liquid remained stationary at this point and rotated clockwise, when viewed from above.

With further increase to 10°, the vortex band formed slightly nearer the leading edge and extended nearer to the root. At the tip, the small patch previously noted became more expansive, and trails of liquid round it indicated clockwise rotation over a larger region. The flow along the wing tip curve was now in two distinct parts; forward, it was towards the leading edge and aft, it was towards the trailing edge. At mid-semi-span, there were signs of a slightly unsteady rearward breaking of the spanwise band of liquid.

At 11° there was a marked change in the pattern. The vortex formed along the leading edge from the root outboard, but about mid-semi-span, it turned round abruptly and an intense band of liquid flowed steadily along it chordwise towards the trailing edge. Just outboard of this change of direction, the leading edge was scrubbed clean, but slightly farther outboard a new vortex, much narrower and closer to the leading edge, could be seen to grow.

Figure 3 is a close-up view of the vortex at the point where it swings aft. The faint lines in the photograph are 0.025 chord apart, so it will be seen that the bubble is about this width, but narrows to about 0.01 chord before it turns. Just outboard, the smooth deposit of lamp-black suggests normal transition, and beyond this, the beginnings of a new vortex can be discerned. This is extremely narrow, appearing as a slightly darker thin line.

At the trailing edge, the chordwise vortex turned and fluid flowed along the edge out to the tip. Most unexpectedly, it did not collect there, but turned and flowed **forwards** in a narrow band towards the leading edge. It carried on inwards along the leading edge to accumulate in a marked vortex at the junction with the outward flow along the leading edge. The main portion of the boundary layer between the chordwise vortex and the tip flowed outwards and then swung round, as sketched. In the photographs, this region always appears almost clear of lamp-black, because the scrubbing action of the flow seems to be very powerful, so that fluid is either swept aft to feed the flow along the trailing edge, or forward to feed the trail coming along the tip.

At 12° the pattern exhibits the same features, but the chordwise flow now starts farther inboard and towards the trailing edge the vortex curves outboard more to flow along it. Instead of flowing right to the tip, it starts swinging forward at about 0.9 semi-span, and similarly, instead of following the wing tip curve, it slopes more sharply inboard to meet the "standing" vortex on the leading edge. This has also moved farther inboard to about 0.8 semi-span.

With further increase of incidence, the features become more marked. The leading edge "standing vortex," which is now indicated by an accumulation of liquid of about ½ inch diameter rotating very vigorously, moves closer and closer inboard until at 18° it is about 0.45 semi-span. The curvature of the flow from the trailing edge starts about mid-semi-span, and has completely turned inboard within 0.8 semi-span. A column of liquid streams in from the extreme rear tip to feed this main stream and flows into the standing vortex. There is still a small bubble vortex parallel to the leading edge, but it extends only to about 0.2 semi-span before being swept round aft. Trails of fluid feeding into the main stream outboard of the standing vortex indicate clearly the nature of the flow surrounding it.

The effect of boundary layer fences, as fitted to some aircraft, has been studied. In the light of the flow patterns shown above, the fences would not be expected to influence the stalling behaviour of the wing. Fig. 4 is a typical example with two nose fences extending two-thirds of the chord aft. It will be seen that there are two strong "standing" vortices formed on the leading edge outboard of each fence, the main flow along the trailing edge appearing to divide to flow forward in both regions of the wing.

Inboard of each fence, there is formed a strong "standing" vortex. The growth of the "standing" vortex at the leading edge tip, and its subsequent movement inboards, is not affected by the presence of the fences.

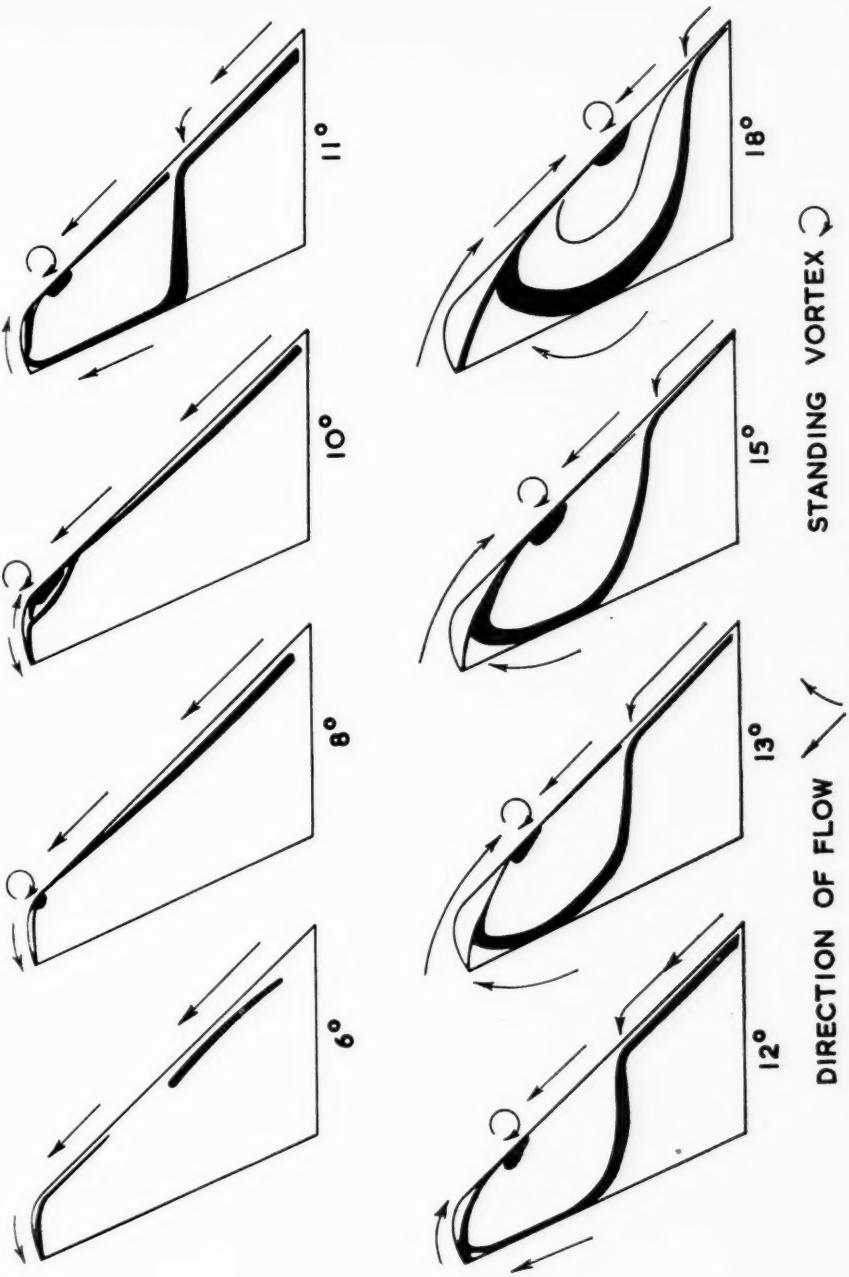

Fig. 1. Boundary layer lamp-black patterns.

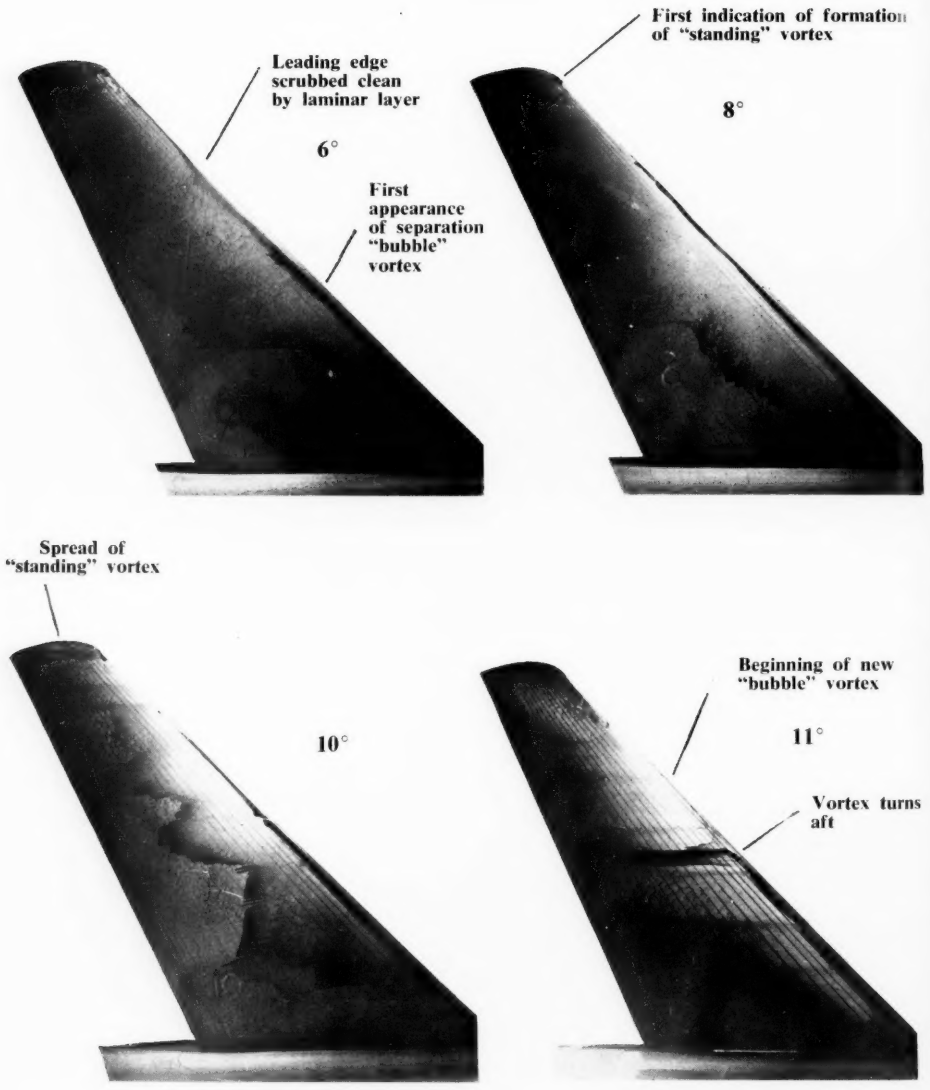

Fig. 2. Boundary layer lamp-black patterns.

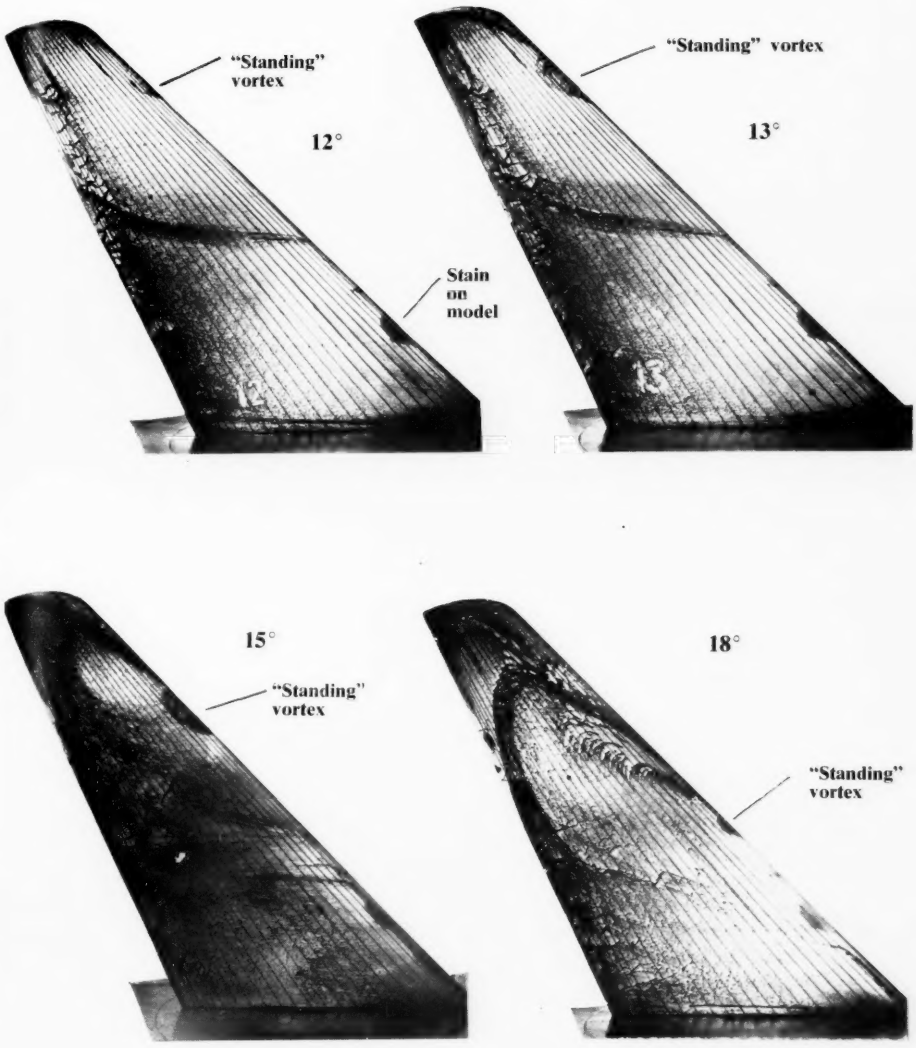

Fig. 2. Boundary layer lamp-black patterns.

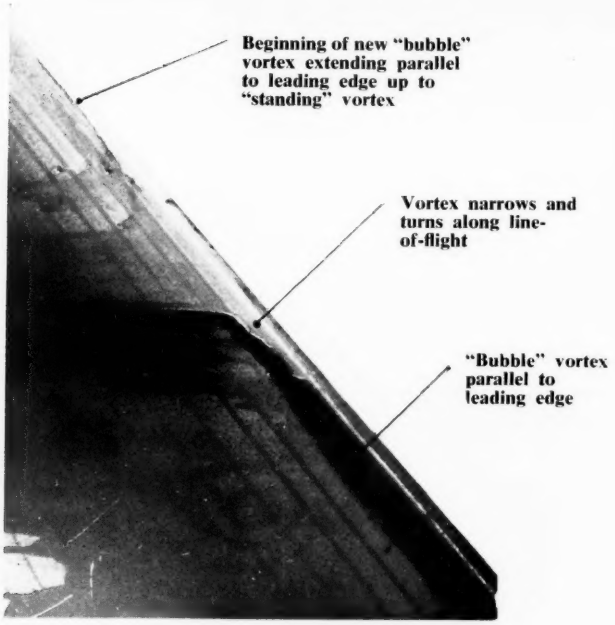
Beginning of new "bubble" vortex extending parallel to leading edge up to "standing" vortex
Vortex narrows and turns along line-of-flight
"Bubble" vortex parallel to leading edge

Fig. 3. Close-up view of separation "bubble" vortex where it turns along the line-of-flight direction.

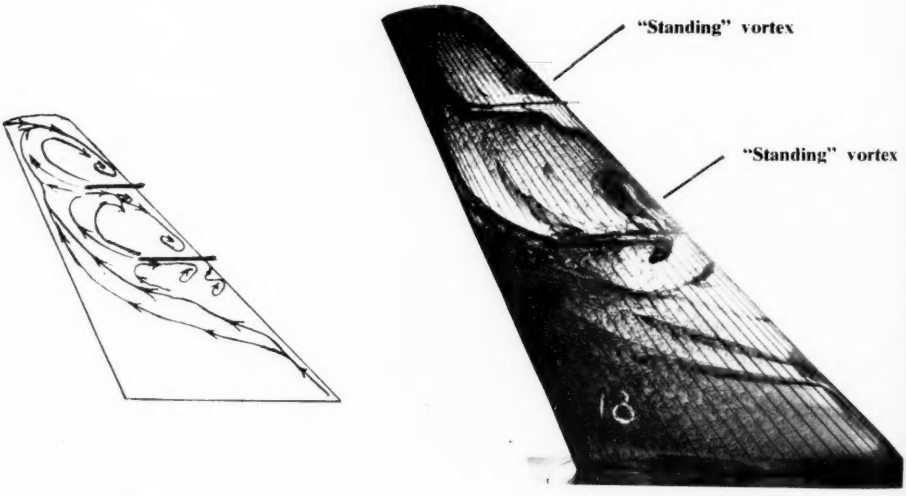

Fig. 4. Boundary layer pattern with two $\frac{2}{3}$ chord nose fences fitted. $\alpha = 18°$.

3. CONCLUSIONS

The behaviour of the boundary layer flow on swept-back wings using thin aerofoil sections with small nose-radius is conditioned largely by the formation of a bubble vortex under the separated laminar boundary layer at the nose. At low incidences, this vortex lies parallel and close to the leading edge along the whole span, and there is a strong spanwise flow along it towards the tip.

At higher incidences, the vortex curves abruptly rearwards about mid-span, in the case of the wing tested, and extends chordwise almost to the trailing edge. The incidence at which this break occurs is extremely critical, a $\frac{1}{4}^\circ$ change either way causing the vortex to form in one direction or the other. The flow along the vortex, after crossing the wing, travels the length of the trailing edge, although some vorticity may be detectable in the free stream aft of the trailing edge where the vortex first meets it; the greater part of the vortex strength is confined to the flow on the wing surface. From the tip, the flow goes forward to meet a strong vortex which forms inboard on the leading edge, and remains stationary at any particular incidence. With increasing incidence, this "standing" vortex forms farther and farther inboard, and the main flow similarly curves round to flow into it from closer inboard.

Although these results were obtained at a comparatively low Reynolds number, they are believed to be relatively independent of Reynolds number, since they originate with laminar separation.

REFERENCES

1. GAULT, D. E. (1949). Boundary Layer and Stalling Characteristics of the N.A.C.A. 63-009 Airfoil Section. N.A.C.A. Technical Note 1894, June 1949.

2. MCCULLOUGH, G. B. and GAULT, D. E. (1949). Boundary Layer and Stalling Characteristics of the N.A.C.A. 64 A 006 Airfoil Section. N.A.C.A. Technical Note 1923, August 1949.

THE ROYAL AERONAUTICAL SOCIETY

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

EIGHTY-SEVENTH ANNUAL REPORT OF THE COUNCIL 1951–1952

The Council for the year 1951-1952 was as follows:—

COUNCIL 1951-1952

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J. G. ROXBURGH (*Graduate*)
W. TYE, O.B.E., B.Sc. (*Fellow*)
C. F. UWINS, O.B.E., A.F.C. (*Fellow*)

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Honorary Treasurer: C. F. UWINS, O.B.E., A.F.C. (*Fellow*)
Honorary Librarian: J. E. HODGSON (*Honorary Fellow*)
Solicitor: L. A. WINGFIELD, M.C., D.F.C. (*Associate*)
Secretary: CAPTAIN J. L. PRITCHARD, C.B.E. (*Honorary Fellow*) until June 1951
DR. A. M. BALLANTYNE, T.D. (*Associate Fellow*)

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D. B. SMITH, A.F.R.Ae.S.
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M. LANGLEY, F.R.Ae.S.
Group Capt. W. G. MATTHEWS (representing Air Vice-Marshal A. H. ROBSON)
D. J. FARRAR, A.F.R.Ae.S.
Dr. E. S. MOULT, F.R.Ae.S.
E. F. RELF, F.R.Ae.S.

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Sir JOHN S. BUCHANAN, F.R.Ae.S.
Sir ARTHUR GOUGE, F.R.Ae.S.

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G. H. DOWTY, F.R.Ae.S.
Sir ARTHUR GOUGE, F.R.Ae.S.
Professor A. A. HALL, F.R.Ae.S.
P. G. MASEFIELD, F.R.Ae.S.
Captain J. L. PRITCHARD, Hon.F.R.Ae.S.
S. CAMM, F.R.Ae.S.

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S. CAMM, F.R.Ae.S.
A. C. CLINTON, F.R.Ae.S.
W. TYE, F.R.Ae.S.
H. SAMMONS, F.R.Ae.S.
E. T. JONES, F.R.Ae.S.
Dr. P. B. WALKER, F.R.Ae.S.
B. S. SHENSTONE, F.R.Ae.S.

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M. C. CAMPION, Grad.R.Ae.S.
A. V. CLEAVER, A.F.R.Ae.S.
E. J. CATCHPOLE, Grad.R.Ae.S.
J. R. COWNIE, Stud.R.Ae.S.
J. C. HALL, Stud.R.Ae.S.
J. W. F. HOUSEGO, Grad.R.Ae.S.
F. G. IRVING, Grad.R.Ae.S.
R. PRIZEMAN, Stud.R.Ae.S.
S. SCOTT HALL, F.R.Ae.S.

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Air Commodore F. R. BANKS, F.R.Ae.S.
Wing Commander C. G. BURGE, A.R.Ae.S.
L. BRIDGMAN, A.R.Ae.S.
Major R. H. MAYO, F.R.Ae.S.
W. S. FARREN, F.R.Ae.S.

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S. SCOTT HALL, F.R.Ae.S.
M. B. MORGAN, F.R.Ae.S.
A. V. CLEAVER, A.F.R.Ae.S.
P. G. MASEFIELD, F.R.Ae.S.
D. L. HOLLIS WILLIAMS, F.R.Ae.S.
J. SMITH, F.R.Ae.S.
N. J. HANCOCK, A.F.R.Ae.S.

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N. E. ROWE, F.R.Ae.S.
S. CAMM, F.R.Ae.S.
W. S. FARREN, F.R.Ae.S.
W. E. W. PETTER, F.R.Ae.S.
Dr. E. S. MOULT, F.R.Ae.S.

REPRESENTATIVES ON OTHER BODIES

General Board of the National Physical Laboratory: Sir Frederick Handley Page and Major G. P. Bulman.

Executive Board of the National Physical Laboratory: Sir Arthur Gouge.

Royal Society—National Committee for Theoretical and Applied Mechanics: Professor A. A. Hall.

Institution of Electrical Engineers—Committee on Regulations for the Electrical Equipment of Aircraft: Mr. C. G. A. Woodford.

Institution of Electrical Engineers—Committee on Radio Equipment for Civil Aircraft: Mr. C. H. Jackson.

Institution of Mechanical Engineers—National Certificates and Diplomas in Mechanical Engineering: Professor F. T. Hill and Sir Frederick Handley Page.

Engineering Joint Council: Major F. B. Halford.

Engineering Joint Council—Special Education Representative: Professor G. T. R. Hill.

Governing Body of Loughborough College—Mr. A. G. Elliott.

Board of Governors of the R.A.E. Technical College: Mr. W. S. Farren.

College of Aeronautics Board of Governors: Sir A. H. Roy Fedden and Dr. H. Roxbee Cox.

British Standards Institution—Nomenclature Committee: Dr. D. M. A. Leggett.

British Standards Institution—Aircraft Committee: Sir A. H. Roy Fedden and Sir Arthur Gouge.

British Standards Institution—Jewels and Pivots for Instruments Committee: Major B. W. Shilson.

British Standards Institution—Technical Committee M/1: Mr. S. Camm.

Segrave Trophy Committee: Major R. H. Mayo.

City and Guilds of London Institute—Advisory Committee on Aeronautical Engineering Practice: Sir A. H. Roy Fedden, Professor R. L. Lickley, Mr. B. S. Shenstone and Captain J. L. Pritchard.

City and Guilds of London Institute—Education Policy Committee: Sir Arthur Gouge.

Association of Special Libraries and Information Bureaus: Mr. J. E. Hodgson (Honorary Librarian).

National Central Library: Mr. J. E. Hodgson (Honorary Librarian).

University of London—Faculty of Engineering Special Committee: Mr. R. S. Stafford.

Regional Advisory Committee for Mechanical Engineering: Sir John S. Buchanan.

ANNUAL GENERAL MEETING

The Annual General Meeting of the Society, held on Monday, 10th May 1951, was reported in the JOURNAL for December 1951.

FELLOWSHIP OF THE SOCIETY

At the Annual General Meeting of the Society held on 10th May 1951 the following were elected Fellows of the Society:—

A. D. Baxter
H. T. Chapman
L. Howarth
M. S. Kuhring
M. J. O. Lobelle
J. Martin
E. Mensforth
F. W. Page
M. Roy
D. Williams
A. D. Young

LECTURES

During the year 15 Main Lectures were given before the Society of which two were held at the following Branches:—

Coventry Branch—15th February 1951. "Some Aspects of Flight Research" by Handel Davies, M.Sc., F.R.Ae.S.

Cheltenham Branch—22nd November 1951. "Problems of Transonic Flight" by A. N. Clifton, B.Sc., F.R.Ae.S.

This is the second year of Section Lectures, which were started in October 1949. They have proved remarkably successful, have been very well attended and produced lively discussions. The subjects chosen are highly specialised and are usually unsuitable for a normal lecture.

A full list of lectures was published in the JOURNAL for December 1951.

WILBUR WRIGHT MEMORIAL LECTURE

The 39th Wilbur Wright Memorial Lecture was read on Monday, 10th September 1951, by Mr. A. E. Raymond, of the Douglas Aircraft Corporation, California, on "The Well-Tempered Aircraft." The complete report was published in the October 1951 JOURNAL.

LOUIS BLERIOT LECTURE

The Fourth Louis Bleriot Lecture was read in London on 23rd February 1951, by Monsieur Maurice Roy on "Power versus Weight in Aviation." A complete report was published in the May 1951 JOURNAL.

BRITISH COMMONWEALTH AND EMPIRE LECTURE

The Seventh British Commonwealth and Empire Lecture was given on Thursday, 4th October 1951, by Air Vice-Marshal Sir Leonard Isitt, K.B.E., R.N.ZA.F., on "Air Transport in New Zealand and the South Pacific." A complete report was published in the November 1951 JOURNAL.

ELECTION OF PRESIDENT AND VICE-PRESIDENTS

Major F. B. Halford, C.B.E., F.R.Ae.S., was elected President of the Society for 1951-52 and took office at the Annual General Meeting on the 10th May 1951.

The following were elected Vice-Presidents of the Society for 1951-52 at the Meeting of Council held on 24th May 1951:—

S. Camm, C.B.E., F.R.Ae.S.

G. H. Dowty, F.R.Ae.S.

G. R. Edwards, M.B.E., B.Sc., F.R.Ae.S.

MEMBERSHIP

During the past year the total membership of the Society has remained as in the previous year, but the distribution in classes has changed. The Associate Fellows have increased in number, but the number of students and graduates has fallen off.

The Table on page 290 gives the state of the membership at 31st December 1951.

DIVISIONS

The Australian, New Zealand and South African Divisions have continued to maintain their membership.

During the year several members of Council have visited Australia and New Zealand and addressed the Divisions.

At the Third Anglo-American Conference papers were read by representatives of these Divisions.

The Society were pleased to welcome Mr. Isbister, the Honorary Secretary of the Australian Division.

BRANCHES

While Major Bulman was President of the Society he visited twenty Branches, and these visits were very popular and beneficial.

Unfortunately Major Halford, through illness, has not been able to visit any of the Branches during the year, but the Secretary has visited about half of the total number.

Many of the lectures read before the Branches have been given in the summary of the activities of the year in the December JOURNAL.

Two main Society Lectures have been given at Coventry and Cheltenham. The attendances at these have been very good, exceeding the average attendance at a London lecture.

MEMBERSHIP

MEMBERSHIP AT 31st DECEMBER 1951

Grade	London Register	Australian Division	New Zealand Division	Southern Africa Division	Life and Honorary	Totals	Suspended Members (included in foregoing figures)
Fellows ...	352 (350)	13 (12)	— (1)	1 (1)	52 (47)	418 (411)	2 (2)
Associate Fellows	2686 (2611)	90 (71)	27 (30)	38 (42)	16 (12)	2857 (2766)	83 (89)
Associates ...	1951 (1994)	67 (71)	50 (51)	87 (91)	5 (4)	2160 (2211)	139 (164)
Graduates ...	1195 (1177)	41 (31)	5 (5)	4 (2)	— (—)	1245 (1215)	43 (37)
Students ...	714 (761)	52 (53)	1 (—)	2 (2)	— (—)	769 (816)	57 (84)
Founder Members	14 (16)	— (—)	— (—)	— (—)	1 (1)	15 (17)	4 (2)
Companions...	152 (167)	6 (2)	3 (1)	5 (5)	11 (11)	177 (186)	10 (18)
Temp. Hon. Members	— (—)	— (—)	— (—)	— (—)	43 (41)	43 (41)	— (—)
	7064 (7076)	269 (240)	86 (88)	137 (143)	128 (116)	7684 (7663)	338 (396)

Figures in brackets are corresponding figures at 31st December 1950

GRADUATES' AND STUDENTS' SECTION

The Graduates' and Students' Section has held eight meetings during the year at which the average attendance was 43. In addition two visits have been paid to aircraft factories. In November the Section held a very successful reception and dance at the Society.

This is the second year in which the Section's affairs have been administered by a Committee of Council and the arrangement has continued to function very satisfactorily. Three members of the Section were co-opted onto the Committee during the year in order to permit wider delegation of the Committee's voluntary work. The names of the Committee are given in the list of Committees at the beginning of this report.

The Committee has made special efforts this year to encourage Graduates and Students to deliver lectures to the Section and these have met with some success in that all but one of the lectures for the Section's 1952 autumn programme will be given by Graduates.

HONORARY FELLOWSHIP

The following were awarded Honorary Fellowship of the Society in 1951:—

Professor Sir Bennett Melvill Jones
Sir Henry Tizard
Mr. C. C. Walker

HONOURS AWARDED TO MEMBERS

The following members of the Society received awards in 1951:—

NEW YEAR HONOURS—JANUARY 1951

Order of Merit	Marshal of the Royal Air Force The Rt. Hon. Hugh Viscount Trenchard (*Honorary Fellow*)
G.C.B.	Air Chief Marshal Sir James Milne Robb (*Associate Fellow*)
K.B.E.	H. M. Garner (*Fellow*)
C.B.E.	Acting Air Vice-Marshal T. N. McEvoy (*Associate Fellow*) H. Constant (*Fellow*) J. E. Serby (*Fellow*)
M.B.E.	P. R. Allison (*Associate Fellow*) J. C. C. Taylor (*Associate*)
A.F.C.	Squadron Leader R. A. Watts (*Associate Fellow*)

BIRTHDAY HONOURS—JUNE 1951

C.B.E.	H. T. Chapman (*Fellow*) G. W. H. Gardner (*Fellow*) W. E. W. Petter (*Fellow*)
O.B.E.	H. E. Hancocke (*Associate Fellow*)
A.F.C.	Squadron Leader A. E. Callard (*Associate*) Commander (E.) P. S. Wilson (*Associate Fellow*)
M.B.E.	G. Maclaren Humphreys (*Associate Fellow*)

INSTITUTE OF THE AERONAUTICAL SCIENCES

Sir Henry Tizard, *Fellow*, was awarded Honorary Fellowship of the Institute of the Aeronautical Sciences. Only one such award is made in the year to a non-American.

WRIGHT BROTHERS' MEMORIAL TROPHY

Professor J. C. Hunsaker, *Honorary Fellow*, was awarded the Wright Brothers' Memorial Trophy for 1951 in appreciation of his long career in aviation public service.

WRIGHT BROTHERS' MEDAL

Mr. James C. Floyd, *Associate Fellow*, was awarded the Wright Brothers' Medal of the Society of Automotive Engineers.

THE DANIEL GUGGENHEIM MEDAL

The Guggenheim Medal for 1950 was awarded to Dr. H. L. Dryden, *Fellow*, for his outstanding leadership in aeronautical research and fundamental contributions to aeronautical science.

For 1951 it was awarded to Mr. Igor Sikorsky "for a lifetime of outstanding contributions to aeronautics, including pioneering with multi-engine aeroplanes, flying boats, amphibians and helicopters."

THE GOLD MEDAL OF THE F.A.I.

The Gold Medal of the Féderation Aéronautique Internationale was awarded to Air Commodore Sir Frank Whittle, K.B.E., C.B., C.B.E., M.A., D.Sc., F.R.S., *Fellow*.

INSTITUTE OF NAVIGATION GOLD MEDAL

The Gold Medal for 1950-51 of the Institute of Navigation was awarded to Mr. H. C. Pritchard, *Fellow*, for valuable contributions in the field of airborne navigation.

THE KELVIN MEDAL

The Kelvin Medal, 1950, was presented to Dr. Theodore von Kármán, *Honorary Fellow*, of the California Institute of Technology, by the Institution of Civil Engineers, on Tuesday, 6th November 1951.

CITY AND GUILDS INSTITUTE

Mr. S. Scott Hall, C.B., M.Sc., D.I.C., *Fellow*, was awarded Fellowship of the City and Guilds Institute.

GUILD OF AIR PILOTS AND AIR NAVIGATORS OF THE BRITISH EMPIRE

Mr. J. Lankester Parker, O.B.E., *Fellow*, was elected Master of the Guild of Air Pilots and Air Navigators on 12th April 1951.

MEDALS AND AWARDS OF THE SOCIETY

Full particulars of the Medals and Awards of the Society were published in the October 1951 JOURNAL. These Medals and Awards were presented at the Thirty-Ninth Wilbur Wright Memorial Lecture on the 10th September 1951.

ELLIOTT MEMORIAL PRIZE

A list of Halton Apprentices awarded the Elliott Memorial Prize was published in the December 1951 JOURNAL of the Society.

JOURNAL PREMIUM AWARDS

A list of Premium Awards made in 1951 was published in the December 1951 JOURNAL of the Society.

ROYAL AERONAUTICAL SOCIETY CHARTER SCHOLARSHIP

The Charter Scholarship for 1951-52 was awarded to Thomas E. B. Bateman, *Graduate.*

THE GEOFFREY DE HAVILLAND MEMORIAL SCHOLARSHIP

The Geoffrey de Havilland Memorial Scholarship for 1951-52 was awarded to Ian Douglas Neilson

THIRD ANGLO-AMERICAN AERONAUTICAL CONFERENCE

A full account of the Third Anglo-American Aeronautical Conference was published in the November 1951 JOURNAL. Negotiations are already under way for the next Conference, to be held in America.

GARDEN PARTY

The Society's Annual Garden Party was held on Sunday, 6th May 1951, and a full account was published in the July 1951 JOURNAL.

OFFICERS AND COMMITTEES

The Council wish to place on record their appreciation of the work which has been done by the Honorary Officers of the Society during the year.

Mr. C. F. Uwins, *Fellow*, the Honorary Treasurer, has continued to advise the Society on its finances with his customary care and attention.

Mr. J. E. Hodgson, Honorary Librarian for so many years, has again during the year continued to advise the Society on all matters pertaining to the Library. This help is particularly appreciated as Mr. Hodgson has not been at all well during the year.

The Council wish to take this opportunity of thanking the members of the various Committees for their useful work on behalf of the Society. These Committees often mean working late hours, and the valuable time given by the various members is greatly appreciated.

ACKNOWLEDGMENTS

Items presented to the Society during the year were acknowledged in the JOURNAL for December 1951.

SOCIETY OF BRITISH AIRCRAFT CONSTRUCTORS—EDUCATIONAL GRANTS

The Joint Selection Committee of the Society of British Aircraft Constructors and the Royal Aeronautical Society awarded Educational Grants to the following successful candidates on 24th July 1951:—

R. E. Catterwell
W. H. Corner
Clive W. Hicks
G. Rampling
J. W. Sowerby

These Grants are for the assistance of young men who are unable, for financial reasons, to obtain training in aeronautical engineering. All those awarded Society of British Aircraft Constructors' Grants are expected to qualify for a technical grade in the Royal Aeronautical Society.

FINANCE

The Income and Expenditure Accounts and Balance Sheets of the Royal Aeronautical Society and Aeronautical Trusts Ltd. for 1951 are published with this Report.

THE ROYAL
(INCORPORATED BY

Balance Sheet

1950 Figures £ s. d.		£ s. d.	£ s. d.
4838 3 11	*Current Liabilities*— Sundry Creditors	5273 1 3	
1901 2 8	Subscriptions and other amounts received in advance	2863 12 11	8136 14 2
6739 6 7			
	Surplus—		
	Publications Development Account—		
	Balance at 31st December 1950	3864 6 1	
	Deduct Expenditure on AERONAUTICAL QUARTERLY, *less* sales, during year	1184 2 3	
3864 6 1		2680 3 10	
1250 0 0	*Reserve for Anglo-American Conference* 1951 ...	— — —	
	Income and Expenditure Account—		
	Balance at 31st December 1950 5001 1 9		
	Less Excess of Expenditure over		
5001 1 9	Income for year to date ... 383 2 6	4617 19 3	7298 3 1
10115 7 10			
£16854 14 5			£15434 17 3

F. B. HALFORD, *President.*
C. F. UWINS, *Honorary Treasurer.*

REPORT OF THE AUDITORS TO THE MEMBERS

In our opinion and to the best of our information and according to the explanations given to us, the above Royal Aeronautical Society Endowment Fund included in the annexed accounts of Aeronautical Trusts Ltd., give a ended on that date.

We have obtained all the information and explanations which to the best of our knowledge and belief were accounts are in agreement therewith.

3 Frederick's Place, Old Jewry, London, E.C.2.
19th March 1952.

Income and Expenditure Account

1950 Figures £ s. d.		£ s. d.	£ s. d.
	To Establishment Charges—		
1882 7 5	Ground Rent, Heating, Lighting, Insurance and Repairs		2059 2 8
	,, Office and Staff Expenses—		
7327 11 2	Salaries	8323 13 4	
328 8 5	Staff Pension Premiums and Pension paid ...	620 0 5	
1536 6 6	Printing and Stationery	1593 6 0	
1151 17 3	Postages and Telephones	1374 6 5	
— — —	Office Equipment	41 12 3	
638 14 7	Other Charges	783 10 3	12736 8 8
10982 17 11			
	,, Journal and Sundry Publications—		
13972 2 6	Printing Costs	16695 12 2	
1856 6 6	Postages	1933 16 7	
1907 2 0	Other Charges	2277 16 9	
17735 11 0		20907 5 6	
4193 7 6	*Less* Sales 5182 17 0		
9649 10 0	Advertisement Revenue ... 9484 10 0	14667 7 0	6239 18 6
3892 13 6			
	,, Expenditure on Data Sheets—		
117 9 2	*Less* amounts recoverable		128 19 2
495 11 8	,, Meetings		596 6 2
	,, Anglo-American Aeronautical Conference 1951		
1250 0 0	(*less* reserve £1250)		4741 4 0
	(NOTE.—No credit has been taken into account in respect of the profit which is expected to be made on the sale of "Conference Proceedings.")		
490 14 1	,, Garden Party		517 0 3
469 14 2	,, Dinners and Receptions		442 0 7
1294 7 11	,, Library Expenses		1223 9 2
326 12 6	,, Branch Expenses		414 0 0
164 9 2	,, Prizes and Donations		127 2 0
300 0 0	,, Charter Scholarship		250 0 0
895 6 10	,, Legal and Professional Charges		293 12 8
477 8 5	(Surplus)		
£23039 12 9			£29769 3 10

APRIL 1952

AERONAUTICAL SOCIETY

ROYAL CHARTER 1949)

31st December 1951

1950 Figures £ s. d.		£ s. d.	£ s. d.
	Current Assets—		
1 0 0	Stock of Journals and other publications ...	1 0 0	
4236 3 10	Sundry Debtors	4927 9 8	
338 18 11	Payments in Advance	129 9 10	
3797 5 6	Cash at Bank and in Hand	576 8 2	5634 7 8
8373 8 3			
7801 4 2	*Investments at Cost* ... ... Market value 31st December 1951 £7,022 (1950 £7,841)		7801 4 2
	Aeronautical Trusts Limited—		
1 1 0	21 shares of 1/- each fully paid at cost	1 1 0	
629 1 0	Amount due on Current Account	1948 4 5	1949 5 5
630 2 0			
	Printed Books, Binding, Old Prints, etc.		
50 0 0	At nominal amount		50 0 0
£16854 14 5			£15434 17 3

OF THE ROYAL AERONAUTICAL SOCIETY

balance sheet and the annexed income and expenditure account of the Society together with the accounts of the true and fair view of the state of the Society's affairs as at 31st December 1951 and of its deficit for the year

necessary for our audit. In our opinion the Society has kept proper books of account and the above mentioned

(Signed) PRICE WATERHOUSE & CO.

for the year ended 31st December 1951

1950 Figures £ s. d.		£ s. d.	£ s. d.
21670 3 3	By Annual Subscriptions		22272 19 6
745 10 6	„ Donations		2074 10 0
124 10 4	„ Interest on Investments (less tax)	121 13 10	
135 7 6	„ Income Tax Recovered	101 17 4	
— — —	„ Surplus on Endowment Fund Income and Expenditure Account for year	4810 9 5	5034 0 7
259 17 10			
	„ Examinations—		
894 6 0	Fees Received	556 0 6	
530 4 10	*Less* Expenses	551 9 3	+ 11 3
364 1 2			
— — —	„ Balance, being excess of Expenditure over Income for year carried to Balance Sheet ...		383 2 6
£23039 12 9			£29769 3 10

AERONAUTICAL

Balance Sheet

1950 Figures										
£	s.	d.		£	s.	d.	£	s.	d.	
			SHARE CAPITAL—							
			Authorised—40 shares of 1s. each				2	0	0	
1	1	0	*Issued*—21 shares of 1s. each fully paid ...							1 1 0
			ROYAL AERONAUTICAL SOCIETY ENDOWMENT FUND—							
			Capital Account—							
131954	8	5	Balance at 31st December 1950				131649	12	1	
1	1	0	*Add*—Donations received during year ...				5	5	0	
927	0	0	Entrance Fees received during year				497	14	0	
132882	9	5					132152	11	1	
1232	17	4								
(Loss on realisation of investments)										
131649	12	1								
			Income Account—							
5799	17	3	Balance as at 31st December 1950 ...				9103	14	6	141256 5 7
3303	17	3								
(surplus)										
140753	6	7								
			PILCHER MEMORIAL FUND—							
			Capital Account—							
99	14	0	As at 31st December 1950 ...				99	14	0	
			Income Account—							
74	7	0	Balance at 31st December 1950	74	9	4				
2	4		*Add*—Income for year to date	5	7	4	79	16	8	179 10 8
(surplus)										
174	3	4								
			USBORNE MEMORIAL FUND—							
			Capital Account—							
109	2	5	As at 31st December 1950 ...				109	2	5	
			Income Account—							
81	12	8	Balance at 31st December 1950	82	7	0				
14	4		*Add*—Income for year to date	5	19	4	88	6	4	197 8 9
(surplus)										
191	9	5								
			HERBERT AKROYD STUART FUND—							
			Capital Account—							
691	9	0	As at 31st December 1950 ...				691	9	0	
			Income Account—							
544	0	5	Balance at 31st December 1950	581	12	3				
			Add—Surplus of Income over							
37	11	10	Expenditure for year to date	16	11	10	598	4	1	1289 13 1
(income)										
1273	1	3								
			R.38 MEMORIAL FUND—							
			Capital Account—							
981	13	10	As at 31st December 1950 ...				981	13	10	
			Income Account—							
869	2	0	Balance at 31st December 1950	928	1	2				
58	19	2	*Add*—Income for year to date	58	19	2	987	0	4	1968 14 2
1909	15	0								
			EDWARD BUSK MEMORIAL FUND—							
			Capital Account—							
449	6	1	As at 31st December 1950 ...				449	6	1	
			Income Account—							
222	17	3	Balance at 31st December 1950	226	7	7				
			Add—Surplus of Income over							
3	10	4	Expenditure for year to date	7	6	2	233	13	9	682 19 10
675	13	8								
144978	10	3	*Forward*							145575 13 1

TRUSTS LIMITED

31st December 1951

1950 Figures												
£	s.	d.		£	s.	d.	£	s.	d.	£	s.	d.
1	1	0	CASH IN HAND							1	1	0
			ROYAL AERONAUTICAL SOCIETY ENDOWMENT FUND—									
			Leasehold Property at cost *less* amounts written off 4, 8 and 9 Hamilton Place									
14185	1	9	At cost	14185	1	9						
5816	1	9	*Less* — Amortisation written off to date	6471	1	9	7714	0	0			
8369	0	0										
			Furniture at cost *less* depreciation									
4570	5	0	At cost	4570	5	0						
2487	15	0	*Less* — Depreciation written off to date	2716	5	0	1854	0	0			
2082	10	0										
119731	6	1	Investments at cost ... ... Market value 31st December 1951, £110,567, including £1,000 2½% Defence Bonds (unquoted) at cost (1950 £113,949)				132065	11	3			
130182	16	1					141633	11	3			
			Current assets—									
375	0	0	Sundry debtors	773	5	0						
10824	11	6	Cash at Bank	797	13	9	1570	18	9			
141382	7	7					143204	10	0			
629	1	0	*Less* — Amount due to Royal Aeronautical Society ...				1948	4	5	141256	5	7
140753	6	7										
			PILCHER MEMORIAL FUND—									
156	5	0	Investment at cost ... ... Market value 31st December 1951, £125 (1950, £145)				156	5	0			
17	18	4	Cash at Bank				23	5	8	179	10	8
174	3	4										
			USBORNE MEMORIAL FUND—									
174	11	1	Investment at cost ... ... Market value 31st December 1951, £139 (1950, £161)				174	11	1			
16	18	4	Cash at Bank				22	17	8	197	8	9
191	9	5										
			HERBERT AKROYD STUART FUND—									
1102	15	8	Investment at cost ... ... Market value 31st December 1951, £870 (1950, £1,012)				1102	15	8			
170	5	7	Cash at Bank				186	17	5	1289	13	1
1273	1	3										
			R.38 MEMORIAL FUND—									
1754	6	2	Investment at cost ... ... Market value 31st December 1951, £1,365 (1950, £1,588)				1754	6	2			
155	8	10	Cash at Bank				214	8	0	1968	14	2
1909	15	0										
			EDWARD BUSK MEMORIAL FUND—									
658	8	8	Investments at cost ... ... Market value 31st December 1951, £591 (1950, £691)				658	8	8			
17	5	0	Cash at Bank				24	11	2	682	19	10
675	13	8										
144978	10	3	*Forward*							145575	13	1

AERONAUTICAL

Balance Sheet

1950 Figures £ s. d.		£ s. d.	£ s. d.	£ s. d.
144978 10 3	*Forward*			145575 13 1
	EDWARD BUSK STUDENTSHIP IN AERONAUTICS—			
	Capital Account—			
	Investments and Cash taken over from previous Trustees			
— — —	*less* legal costs of transfer ...			5418 12 7
	WILBUR WRIGHT MEMORIAL FUND—			
	Capital Account—			
2136 17 11	As at 31st December 1950 ...		2136 17 11	
	Income Account—			
213 12 6	Balance at 31st December 1950	219 10 10		
	Add—Surplus of Income over			
5 18 4	Expenditure for year to date	5 18 4	225 9 2	2362 7 1
2356 8 9				
	SIMMS GOLD MEDAL FUND—			
	Capital Account—			
527 15 9	As at 31st December 1950 ...		527 15 9	
	Income Account—			
64 10 3	Balance at 31st December 1950	65 5 11		
	Add—Surplus of Income over			
15 8	Expenditure for year to date	8 3 10	73 9 9	601 5 6
593 1 8				
	ALSTON MEMORIAL FUND—			
	Capital Account—			
245 6 0	As at 31st December 1950 ...		245 6 0	
	Income Account—			
69 12 7	Balance at 31st December 1950	73 10 9		
	Add—Surplus of Income over			
3 18 2	Expenditure for year to date	3 18 2	77 8 11	322 14 11
318 16 9				
	GEOFFREY DE HAVILLAND MEMORIAL FUND—			
	Capital Account—			
4175 17 5	As at 31st December 1950 ...		4175 17 5	
	Income Account—			
133 9 0	Balance at 31st December 1950	242 1 0		
	Add—Surplus of Income over			
108 12 0	Expenditure for year to date	83 3 8	325 4 8	4501 2 1
4417 18 5				
£152664 15 10	F. B. HALFORD, *President.*			£158781 15 3
	C. F. UWINS, *Honorary Treasurer.*			

REPORT OF THE AUDITORS TO THE

In our opinion and to the best of our information and according to the explanations given to us the above Company's affairs and of the Funds administered by it as at 31st December 1951 and of the surplus of the Funds We have obtained all the information and explanations which to the best of our knowledge and belief were accounts, which are in agreement therewith, give in the prescribed manner the information required by the Companies

3 Frederick's Place, Old Jewry, London, E.C.2.
19th March 1952.

TRUSTS LIMITED—*Continued*

31st December 1951

1950 Figures £ s. d.		£ s. d.	£ s. d.	£ s. d.
144978 10 3	*Forward*			145575 13 1
	EDWARD BUSK STUDENTSHIP IN AERONAUTICS—			
	Investments at market value 31st Dec. 1951 ...		4906 10 0	
	Cash at Bank		415 2 6	
— — —	Cash in hands of Solicitors		158 15 1	
			5480 7 7	
	Less—Sundry creditors		61 15 0	5418 12 7
	WILBUR WRIGHT MEMORIAL FUND—			
2250 14 9	Investments at cost		2250 14 9	
	Market value 31st December 1951, £2,070 (1950, £2,405)			
105 14 0	Cash at Bank		111 12 4	2362 7 1
2356 8 9				
	SIMMS GOLD MEDAL FUND—			
504 12 0	Investments at cost		504 12 0	
	Market value 31st December 1951, £414 (1950, £476)			
88 9 8	Cash at Bank		96 13 6	601 5 6
593 1 8				
	ALSTON MEMORIAL FUND—			
290 0 0	Investments at cost		290 0 0	
	Market value 31st December 1951, £285 (1950, £299)			
28 16 9	Cash at Bank		32 14 11	322 14 11
318 16 9				
	GEOFFREY DE HAVILLAND MEMORIAL FUND—			
4175 17 5	Investment at cost		4175 17 5	
	Market value 31st December 1951, £3,407			
242 1 0	(1950, £3,923)			
	Cash at Bank		325 4 8	4501 2 1
4417 18 5				
£152664 15 10				£158781 15 3

MEMBERS OF AERONAUTICAL TRUSTS LTD.

balance sheet and the annexed income and expenditure account give a true and fair view of the state of the for the year ended on that date.
necessary for our audit. In our opinion the company has kept proper books of account and the above mentioned Act, 1948.

(Signed) PRICE WATERHOUSE & CO.

AERONAUTICAL

Income and Expenditure Accounts

ROYAL AERONAUTICAL SOCIETY

1950 Figures				1951		
£	s.	d.		£	s.	d.
485	5	3	To Amortisation of Leasehold Property	655	0	0
228	10	0	„ Depreciation of Furniture	228	10	0
5	5	0	„ Legal Expenses	10	10	0
3303	17	3	„ Surplus of Income over Expenditure for year transferred to Royal Aeronautical Society	4810	9	5
£4022	17	6		£5704	9	5

PILCHER

£	s.	d.		£	s.	d.
5 (award)	5	0	To Income for year carried to Balance Sheet	5	7	4
2 (surplus of income)		4				
£5	7	4		£5	7	4

USBORNE

£	s.	d.		£	s.	d.
5 (award)	5	0	To Income for year carried to Balance Sheet	5	19	4
14 (surplus of income)		4				
£5	19	4		£5	19	4

HERBERT AKROYD

£	s.	d.		£	s.	d.
—	—	—	To 1951 Award	21	0	0
37 (income)	11	10	„ Surplus of Income over Expenditure for year carried to Balance Sheet	16	11	10
£37	11	10		£37	11	10

R.38

£	s.	d.		£	s.	d.
58	19	2	To Income for year carried to Balance Sheet	58	19	2
£58	19	2		£58	19	2

EDWARD BUSK

£	s.	d.		£	s.	d.
21	0	0	To 1951 Award	21	0	0
3	10	4	„ Surplus of Income over Expenditure for year carried to Balance Sheet	7	6	2
£24	10	4		£28	6	2

WILBUR WRIGHT

£	s.	d.		£	s.	d.
75	0	0	To 1951 Award	75	0	0
5	18	4	„ Surplus of Income over Expenditure for year carried to Balance Sheet	5	18	4
£80	18	4		£80	18	4

TRUSTS LIMITED

for the Year ended 31st December 1951

ENDOWMENT FUND

1950 Figures					
£	s.	d.		£	s. d.
2642	1	4	By Interest on Investments (Gross)	2642	1 4
477	7	8	„ Interest on Investments (*less* tax)	815	11 8
206	6	9	„ Refund of Income Tax	736	6 9
75	10	3	„ Interest on Deposit Account	10	9 8
621	11	6	„ Rents receivable	1500	0 0
£4022	17	6		£5704	9 5

MEMORIAL FUND

£	s.	d.		£	s. d.
5	7	4	By Interest on Investments (Gross)	5	7 4
£5	7	4		£5	7 4

MEMORIAL FUND

£	s.	d.		£	s. d.
5	19	4	By Interest on Investments (Gross)	5	19 4
£5	19	4		£5	19 4

STUART FUND

£	s.	d.		£	s. d.
37	11	10	By Interest on Investments (Gross)	37	11 10
£37	11	10		£37	11 10

MEMORIAL FUND

£	s.	d.		£	s. d.
58	19	2	By Interest on Investments (Gross)	58	19 2
£58	19	2		£58	19 2

MEMORIAL FUND

£	s.	d.		£	s. d.
15	17	0	By Interest on Investments (Gross)	15	17 0
4	15	4	„ Interest on Investments (*less* tax)	4	13 2
3	18	0	„ Refund of Income Tax	7	16 0
£24	10	4		£28	6 2

MEMORIAL FUND

£	s.	d.		£	s. d.
80	18	4	By Interest on Investments (Gross)	80	18 4
£80	18	4		£80	18 4

AERONAUTICAL.

Income and Expenditure Accounts

SIMMS GOLD

1950 Figures					
£	s.	d.		£	s. d.
15	11	8	To 1951 Award	8	3 6
	15	8	„ Surplus of Income over Expenditure for year carried to Balance Sheet	8	3 10
£16	7	4		£16	7 4

ALSTON

£	s.	d.		£	s. d.
5	0	0	To 1951 Award	5	0 0
3	18	2	„ Surplus of Income over Expenditure for year carried to Balance Sheet	3	18 2
£8	18	2		£8	18 2

GEOFFREY de HAVILLAND

£	s.	d.		£	s. d.
40	0	0	To 1951 Award	120	0 0
108	12	0	„ Surplus of Income over Expenditure for year carried to Balance Sheet	83	3 8
£148	12	0		£203	3 8

TRUSTS LIMITED—*Continued*

for the Year ended 31st December 1951

MEDAL FUND

1950 Figures				
£ s. d.			£ s. d.	
16 7 4	By Interest on Investments (Gross)		16 7 4	
£16 7 4			£16 7 4	

MEMORIAL FUND

£ s. d.		£ s. d.
8 18 2	By Interest on Investments (Gross)	8 18 2
£8 18 2		£8 18 2

MEMORIAL FUND

£ s. d.		£ s. d.
66 14 6 (less tax)	By Interest on Investments (Gross)	121 6 2
81 17 6	„ Refund of Income Tax	81 17 6
£148 12 0		£203 3 8

DONATIONS

The Council acknowledge with grateful thanks donations from the Blackburn and General Aircraft Ltd., the Bristol Aeroplane Co. Ltd., the de Havilland Aircraft Co. Ltd., Dowty Equipment Ltd., Sir George Nelson, Sir Frederick Handley Page, Rolls-Royce Ltd., the Society of British Aircraft Constructors and Vickers-Armstrongs Ltd.

They also wish to acknowledge contributions from the Ministry of Supply and the Society of British Aircraft Constructors for the work of the Technical Committees of the Society.

LIBRARY

During the year the Library acquired a number of useful text books, and books on aeronautics generally, for the use of members. A list of additions to the Library is published in the JOURNAL each month.

It has been found during the year that increasing use is being made of the information facilities available from the Library and the Society generally.

STAFF

Captain J. L. Pritchard retired from the Secretaryship of the Society in June 1951 and Dr. A. M. Ballantyne, B.Sc., T.D., A.F.R.Ae.S., A.M.I.C.E., A.M.I.Struct.E., took office in July, 1951. Particulars of Dr. Ballantyne's career were given in the October 1951 JOURNAL.

During the year the staff has been increased. The Technical staff is being expanded and should make a very useful team to carry out the Society's technical work.

The Council wish to place on record their appreciation of the extra work undertaken by the staff in the organisation of the very successful Anglo-American Conference held in September 1951. They also wish to record their appreciation of the help and loyal support given by the staff to the new Secretary.

GENERAL

During the year the Society's Committee Rooms have been used regularly by the Aeronautical Research Council, British Standards Institution, Society of British Aircraft Constructors, the Helicopter Association, and many others engaged in aeronautical work.

REVIEWS

The following are reviewed:—

Adhesives for Wood
Analysis of the Four-Bar Linkage
Vibration and Shock Isolation
The Algebra of Vectors and Matrices
Production Engineering Administration and Management
Gas Turbine Theory
Theory of Elasticity (*New Edition*)
Heat and Thermodynamics (*Third Edition*)
Baughman's Aviation Dictionary and Reference Guide (*Third Edition*)
Crispin's Day (*Fiction*)

ADHESIVES FOR WOOD. R. A. G. Knight. Chapman & Hall, London, 1952. 242 pp. Illustrations and Tables. Index. 25s. net. (The third of a series of monographs on metallic and other materials published under the authority of the Royal Aeronautical Society.)

Mr. Knight's reputation as a recognised authority on the gluing of wood entitles one to expect the high standard which this book attains. "Adhesives for Wood" is perhaps an inadequate title, as the contents cover much more than the name implies. There are twenty-two chapters; the first four are of an introductory nature, the next ten chapters deal with gluing technique, followed by three chapters on conditions of use and three on testing; the remaining two chapters deal respectively with the durability of glues and the gluing of wood to other materials.

The behaviour and performance of the synthetic resin adhesives occupy most of the space, but the natural glues rightly receive an honourable mention, although their shortcomings when judged by present-day standards are recognised.

At a time when so much scientific and technical literature is little more than the fruits of diligent abstracting, it is particularly satisfying to read a book which reports so much work initiated and supervised by the author; indeed, one continually senses the intimate touch.

"Adhesives for Wood" is written in a style which makes easy reading. It is well referenced and suggestions are included for the deeper study of certain relevant aspects which are outside the scope of the work, for example, the meaning of pH. Much thought has clearly been given to the glossary of terms at the end of the book, and on the whole the definitions are sufficiently comprehensive.

The reviewers have found few misprints and no statements of fact to which they can take exception. However, the suggestion on p. 46 that melamine formaldehyde resins are suitable as cold-setting adhesives is misleading; and while not contesting the accuracy of the observations made on the relative durability of different glues, the reviewers cannot themselves agree with all the conclusions which Mr. Knight has drawn from those observations. Having said this, however, it should be emphasised that the book is one which will surely become a standard work and one which anyone interested in the subject of glues and gluing should possess.—C.A.A.R. and N.A. DE B.

ANALYSIS OF THE FOUR-BAR LINKAGE. John A. Hrones and George R. Nelson. Chapman & Hall, London, 1951. 750 pp. Illustrated. £6 net.

The Four-Bar Linkage is the oldest and physically the simplest mechanism, yet its theory, as with so many simple things, is long and unwieldy. In aeronautics, the linkage has many applications in the design of controls, servo-mechanisms and undercarriages in addition to its use in the machine shop for the generation of special shapes and for quick return motions. This volume is intended to assist the designer in the selection of a suitably proportioned linkage to generate any particular motion. It has been compiled as an encyclopedia showing diagrammatically the motion of points attached to the connecting rods of various proportions. There are in all 730 full-page diagrams (12 in. by 17 in.) representing the motion of these points and 14 pages of descriptive matter of which about half is composed of illustrations. The full-page diagrams are intended to be worked from directly and the quality of the printing is beyond reproach. Examination of the diagrams shows that an astonishingly large number of arrangements of the mechanism generate shapes looking very much like conventional aerofoils and one wonders that this device is not used extensively in the machining of blading for turbines.

VIBRATION AND SHOCK ISOLATION. Charles E. Crede. John Wiley & Sons Inc., New York. Chapman & Hall, London, 1951. 324 pp. 169 diagrams. Index. 52s. net.

The design and application of isolators or flexible mountings is covered from the point of view of the designer. The author has had extensive experience in this field in American Industry and the Navy Department. The chapters in the book are as follows:—

1. Introductory and Revision.
2. Vibration Isolation.
3. Shock Isolation.
4. Non-linear systems and damping.
5. Materials—rubber, felt, cork, helical springs.
6. Applications—low speed machinery, presses, looms. Military Equipment—general, aircraft, vehicles, naval vessels and portable equipment.

To illustrate the range of the book, Chapter 2 deals with a single-degree-of-freedom system leading to transmissibility, multi-degree-of-freedom systems with two planes of symmetry, inclined isolators and de-coupling of modes. An extensive field is covered in a general manner and the book will be a great help to the designer who deals with the usual mounting problems.

The general derivation of the well-established rules used in isolator or flexible mounting problems is included, with the addition of some interesting items that have been reported by the author and other writers in current American literature. The reader who has specialised knowledge in a particular field may find that the relatively simple treatment and description has led to slight inaccuracies. For aircraft application, an adequate treatment is given of the mounting of instrument panels, radio, radar, bombsights, navigational aids, and so on.

The vast amount of practical experience that has been obtained in America on the mounting of reciprocating and gas turbine propeller engines is briefly summarised. The theoretical treatment of mounting items to a member which cannot be taken to be large and stiff, but which has a dynamic flexibility at the point of connection similar to that of the item being mounted, is not included in full.

We still have to wait for an authoritative comparison between engine mountings using relatively flexible isolators and mountings with stiff items at the connecting points, to know if the former method has any other advantages than greater possibilities in de-coupling of modes and that the value of the flexibility used in the isolator can be readily changed.

THE ALGEBRA OF VECTORS AND MATRICES. Thomas L. Wade. Addison-Wesley Press, Cambridge, Mass., 1951. 89 pp. 32s. 3d. net.

Not very many years ago matrices were rarely used in theoretical mechanics. They were almost unheard of by computers and certainly played no recognisable part in the design of aeroplanes or other structures. At the present day this neglect by engineers of one of the most valuable aids to clear and concise mathematical presentation is being remedied rapidly. This is shown not only by the increasing number of technical papers which are expressed in the language of matrices, but also by the regular appearance of new text books on modern algebra.

Problems concerning the deformations and the oscillations of aeroplane structures generally involve many variables and cumbrous systems of linear equations. A treatment of this type of problem by longhand algebra has several disadvantages. One obvious disadvantage is that it wastes paper. Another and more vital objection is that the true simplicity or generality of an analytical method can easily be obscured by a glut of symbols. Even if the originator of the method is not so misled, the account he gives of it will be irritating to readers who prefer conciseness and find that they have to do the necessary condensation for themselves. The advantages of matrices are also very obvious with numerical applications. A compact scheme of computation is attractive even with simple problems, and the necessary matrix rules can be learnt in a few hours. With the complex structural problems arising at the present day, such as those concerning swept wings and requiring the use of high-speed computing equipment, a knowledge of matrices is indispensable.

The new addition to the literature of vectors and matrices is elementary in scope and is classed by the author as " suitable for use in the first course of a sequence of courses devoted to modern algebraic theories." The book will be useful to those who wish to understand the underlying ideas of modern higher algebra and their relationship to the physical and applied sciences, without becoming too deeply committed to abstract mathematics. A short introductory chapter gives a clear exposition, helped by simple examples, of basic definitions such

as *group*, *field* and *ring*. The remainder of the book is effectively in two parts. The three chapters on vector algebra begin with simple concepts and lead up to vector spaces and subspaces in n dimensions. The remaining seven chapters, which are devoted to matrices, pass progressively from the more elementary properties to transformations and group properties, the characteristic equation, rank, and algebraic forms. The book is well interspersed with numerical examples and simple exercises, and includes a short bibliography.

GAS TURBINE THEORY. Cohen and Rogers. Longmans Green & Co., London, 1951. 291 pp. Illustrated. 30s. net.

This book aims at setting out the basic aerodynamic and thermodynamic theory of the design and performance of gas turbine engines. The book is intended for students and practising engineers newly approaching this subject, and the authors have certainly been wise in limiting their objective, for the result is a useful volume of reasonable size. The subject matter is compassed within nine chapters following the logical layout of dealing first with the basic thermodynamics, thence proceeding to the theory of component design and performance, and finally a chapter showing how the performance of a complete engine may be formulated from a knowledge of the characteristics of its individual components.

The two chapters on the thermodynamics of gas turbine cycles deal first with cycles of ideal processes followed by a treatment of practical cycles in which the component losses are taken into account. The treatment is clear and in sufficient detail to enable the reader to undertake the thermodynamic analysis of the more complex cycles for himself. In this section, the aircraft jet engine deservedly receives special attention, although here greater clarity would have been achieved by using the concepts of thermal and propulsive efficiency and by explaining the true significance of specific fuel consumption when referred to thrust.

The chapters dealing with compressor, combustion chamber and turbine design are prefixed by an earlier one on gas dynamics. The chapter on gas dynamics, with a clear and unambiguous treatment of flow in nozzles, flow in parallel ducts with heating, and compressibility effects generally, furnishes the means of appreciating fully the basic design theory and criteria given in the later chapters. Centrifugal compressor design and performance is dealt with on orthodox lines but little attempt is made to stress the limitations of the design theory from the aerodynamic viewpoint. This fault is remedied in the chapter on axial compressor design and performance where the treatment and date are as up-to-date as could be expected and quite a useful description of wind tunnels and apparatus used for the testing of cascades of aerofoils is given. On combustion chambers, the book brings out well the practical limitations and factors influencing design and this also forms a feature of the chapter on turbines, where a short section on mechanical design accompanies the usual aero-thermodynamic analysis. The concluding chapter of the book brings together the work of the earlier chapters in showing the method of deriving the performance of specific engine designs, and features at the start the non-dimensional method of approach; the value to the student of this approach cannot be over-estimated.

Altogether, apart from a few faults, the authors have succeeded in producing a book that cannot fail to benefit those newly approaching the subject of gas turbine engines. Illustrated by numerous examples, the volume, if read with the care it deserves, provides a good foundation, upon which those studying the aero-thermodynamic theory of design and performance, may well build.

PRODUCTION ENGINEERING ADMINISTRATION AND MANAGEMENT. Professor J. V. Connolly. *Aircraft Engineering* (Bunhill Publications Ltd.), London, 1951. 50 pp. 5s. net.

This little 50-page monograph is a reprint of a series of articles which appeared in *Aircraft Engineering*.

It deals briefly with the problems encountered in running a factory or similar organisation involving management-technical problems. The booklet itself is by no means exhaustive and merely whets one's appetite for more information. The many references to other works and the good bibliography enables the interested reader to follow up any particular line of thought.

Professor Connolly starts at the beginning and deals with the real basic knowledge that Managers should have and which very few in fact do have. He puts special stress on the value of the newer statistical methods of analysing what is being done or could be done, and although many of his arguments are illustrated by aircraft examples, the work is applicable to a much wider field.

In only one place does Professor Connolly appear to dig too deeply into detail, which is rather inconsistent with the wider scope of the book; this is on the economics of tool cutting based on laboratory work at the College of Aeronautics.

The tools of management are described very well indeed, particularly various kinds of measurement (finance-, cost-, dimensional-, quality-, and work-measurement). Under "Work Measurement" he quotes some extremely interesting comparisons between the United States and the United Kingdom on productivity, which can well be read in conjunction with recent Productivity Reports issued by the Anglo-American Council of Productivity.

Professor Connolly's remarks on methods, motion and time study are very much to the point, and he has had personal experience of much of the equipment he describes.

He then deals with the processes of management, and how the job is controlled, planned and budgeted.

His final section is on the *art* of production management, which is an admission of the fact that it is not a cut and dried job. Most of Professor Connolly's remarks are obvious and straightforward, but they have needed to be set down in the way in which he does it. For instance, he states the obvious fact that authority and responsibility are inseparable: to have one without the other gets you nowhere and yet in many organisations there are peculiar half jobs lacking one or other of these vital factors.

Professor Connolly has done industry, and particularly the Aircraft Industry, a very good turn by bringing to our notice the vast amount of literature available. Much of it has been unused simply because we did not know about it. Certainly no technical library in a manufacturing or operating organisation should be without Professor Connolly's booklet, and even Senior Executives ought to be able to afford to have a copy tucked away in their desks.

THEORY OF ELASTICITY. Timoshenko and Goodier. McGraw-Hill. New York 1951. 493 pp. 270 diagrams. 81s. net. (*New Edition.*)

This is a new and enlarged edition of the classic book published in 1934 with the same title and having Timoshenko as sole author, and it achieves the same high standard of lucidity and accuracy.

While there is little in the old edition not given in the new, the latter contains a much fuller treatment of photoelasticity, thermal stresses, and the behaviour of slender bodies in torsion and bending, and now includes sections on strain gauge problems, and finite difference and relaxation methods. Another addition, greatly to be welcomed, is the inclusion of examples for the student to work through.

HEAT AND THERMODYNAMICS. Mark W. Zemansky. McGraw-Hill, London, 1951. 465 pp. with diagrams. 42s. 6d. net. (*Third Edition.*)

This is the third edition of the book first published in 1937. Some of the old material has been rewritten in order to bring it up to date or to make it more rigorous, and some new material has been added. Among the topics that have been discussed more completely are the temperature concept, the three laws of thermodynamics, natural convection, the phase rule, and various experimental methods. The new topics include a discussion of forced convection with application to the single-current heat exchanger, a new treatment of the thermo-couple by a method equivalent to that of Tolman and Fine, a treatment of dielectrics and piezoelectricity, and a complete chapter on the physics of very low temperatures.

In addition there are many other changes which bring the book up to date, including 65 new figures and 60 new problems.

BAUGHMAN'S AVIATION DICTIONARY AND REFERENCE GUIDE. Revised by E. J. Gentle and C. E. Chapel. 653 pp. Illustrations and tables. Aero Publishers Inc., Los Angeles. $7.50. 64s. net (*Third Edition.*)

This Third Edition is a revised version of the Second Edition, published in 1942. New definitions have been added to cover up-to-date information on atomic energy, electronics, guided missiles, helicopters, jet aircraft, meteorology, ordnance, radar, rockets, television and other subjects intimately related to aviation. The new edition omits the sections General Air Traffic Rules, Guides to Studies for Airmen, Miscellaneous C.A.A. Comments and Miscellaneous Directories, which were contained previously; this is because of their tendency to frequent change. Apart from these changes, and the addition of a small section on radar information, the major part of the second edition is retained.

CRISPIN'S DAY. Leigh Howard. Longmans, Green & Co. Ltd. London 1952. 265 pp. 10s. 6d. net. *Fiction.*

It is a pity that the author has chosen one of the most heroic single actions of the war as background for his novel and a cad with no redeeming features as his " hero." After hearing of the breeching of the Mohne and Eder dams, Flying Officer Smith, a member of the R.A.F. Film Unit, suggests in a spirit of beery bravado that he should record the results of the bombing. The first 210 pages attempt to capture the nervous tension preceding such an operation and the actual flight is dismissed in some 50 pages. Through Flying Officer Smith's carelessness his pilot is killed and Smith, with apparently no previous piloting experience, succeeds in crash-landing at base without harm to himself thus denying the reader even that satisfaction. The author served as a member of the R.A.F. Film Unit during the war and the Mosquito aeroplane, the real hero of the story, gave long and faithful service with the Unit.

ADDITIONS TO THE LIBRARY

The following have been added to the Library. Books and papers marked with an asterisk may not be taken out on loan. The titles of pamphlets are printed in italics.

Air Ministry. Meteorological Handbook for Pilots and Navigators. H.M.S.O. 1951.
Arteaga, F. de. Arquitectura de las Grandes Aeronaves de Transporte: El Barco Volante. La Universidad Cordoba. 1951.
Ashkouti, J. A. Aircraft Mechanic's Pocket Manual. 4th edition. Pitman, New York. 1951.
Brimm, D. J. and H. E. Boggess. Aircraft Maintenance. 2nd edition. Pitman, New York. 1950.
Department of Scientific and Industrial Research. The Fire Hazard of Fuelling Aircraft in the Open. H.M.S.O. 1951.
Hilton, W. F. High-Speed Aerodynamics. Longmans Green, New York. 1951.
Holland, M. and I. M. Smith. Architects of Aviation. Duell, Sloan & Pearce, New York. 1951.
Lagerstrom, P. A. and M. E. Graham. *Linearized Theory of Supersonic Control Surfaces.* Douglas Aircraft Company. 1947.
Lovell, B. and J. A. Clegg. Radio Astronomy. Chapman & Hall. 1952.
Nicholson, J. L. Air Transportation Management. John Wiley. 1951.
Nikolsky, A. A. Helicopter Analysis. John Wiley. 1951.
Teichmann, F. K. Airplane Design Manual. 3rd edition. Pitman, New York. 1950.
Vallance, A. and V. L. Doughtie. Design of Machine Members. 3rd edition. McGraw Hill. 1951.

Aeronautical Research Council. Reports and Memoranda.

2487—*Low-speed wind-tunnel tests on the Baynes carrier wing; a tailless glider. J. Trouncer.*
2500—*Interference on a wing due to a body at supersonic speeds. S. Kirkby and A. Robinson.*
2535—*High-speed wind-tunnel tests on models of four single-engined fighters (Spitfire, Spiteful, Attacker and Mustang), Parts* 1-5. *Staff of the R.A.E. High Speed Wind Tunnel.*
2540—*Tests on a Glas II wing without suction in the compressed air wind tunnel. C. Salter, C. J. W. Miles and Miss R. Owen.*
2549—*An example in wing theory at supersonic speed. H. B. Squire.*
2554—*Some data pertaining to the supersonic axial-flow compressor. I. M. Davidson.*

Aeronautical Research Laboratories, Australia.

Reports SM.

175—*The deflection of a thin flat clamped parallelogram plate subjected to uniform normal loading. I. Mirsky.*
177—*The deflection of a thin flat cantilever square plate subjected to uniform normal loading. I. Mirsky.*
181—*The nature of strain-age-embrittlement. C. J. Osborn.*

Structures and Materials Notes.

191—*Properties of strain gauge wires. R. P. Brown and N. L. Svensson.*
193—*A new electric resistance strain gauge for large strain. N. L. Svensson.*
194—*Some fatigue tests on milled 75S aluminium alloy sheet. J. Y. Mann.*

National Aeronautical Establishment, Canada. Laboratory Reports.

13—*Continuous-trace recording system for flight testing an "Orenda" gas turbine. H. W. Wilson.*

Publications Scientifiques et Techniques du Ministère de l'Air, France.

255—*Détermination en soufflerie de la distribution de la circulation le long de l'envergure des ailes par la méthode des fumées. J. Valensi, H. Parigi et M. Rebont.*

Notes Techniques.

44—*Etude d'un champ aérodynamique par les ultrasons combinés à la strioscopie. Mlle. M. Merle.*

National Aeronautical Research Institute, Amsterdam. Reports.

F.79—*Revised methods for routine calculations of laminar and turbulent boundary layers of two-dimensional incompressible flows. Dr. J. A. Zaat.*

F.97—*An approximate theory of the oscillating wing in a compressible subsonic flow for low frequencies. J. R. M. Radok.*

Monografie Scientifiche di Aeronautica, Italy.

11—*Applicazione del metodo di ritz al calcolo della corrente compressibile attorno ad un cilindro circolare. Helga Blank.*

National Advisory Committee for Aeronautics, U.S.A.

Reports.

1010—*A recurrence matrix solution for the dynamic response of aircraft in gusts. J. C. Houbolt.*

1012—*Investigation of the NACA* 4-(5) (08)-03 *and NACA* 4-(10) (08)-03 *two-blade propellers at forward Mach numbers to* 0.725 *to determine the effects of camber and compressibility on performance. J. B. Delano.*

1013—*Effects of wing flexibility and variable air lift upon wing bending moments during landing impacts of a small seaplane. K. F. Merten and E. B. Beck.*

Technical Notes.

2459—*Some effects of varying the damping in pitch and roll on the flying qualities of a small single-rotor helicopter. J. P. Reeder and J. B. Whitten.*

2556—*Buckling of rectangular sandwich plates subjected to edgewise compression with loaded edges simply supported and unloaded edges clamped. Kuo Tai Yen, V. L. Salerno and N. J. Hoff.*

2570—*Comparison of airspeed calibrations evaluated by the accelerometer and radar methods. L. J. Lina and J. P. Trant, Jun.*

2582—*General consideration of problems in compressible flow using the hodograph method. Chieh-Chien Chang.*

2585—*Calculation of aerodynamic forces on a propeller in pitch or yaw. J. L. Crigler and J. Gilman, Jun.*

2586—*Fundamental effects of cold-work on some cobalt-chromium-nickel-iron base creep-resistant alloys. D. N. Frey, J. W. Freeman and A. E. White.*

2590—*Calculations on the forces and moments for an oscillating wing-aileron combination in two-dimensional potential flow at sonic speed. H. C. Nelson and J. H. Berman.*

2591—*The effects of Reynolds number on the application of NACA* 16*-series airfoil characteristics to propeller design. H. E. Cleary.*

2592—*Orientation of orifices on bodies of revolution for determination of stream static pressure at supersonic speeds. M. Cooper and C. V. Hamilton.*

2595—*Design of two-dimensional channels with prescribed velocity distributions along the channel walls. II—solution by Green's function. J. D. Stanitz.*

Technical Memorandum.

1325—*Concerning the flow about ring-shaped cowlings of finite thickness. Part I. D. Kuchemann.*

Civil Aeronautics Administration, U.S.A. Technical Development Report.

150—*A Study of the visibility and glare ranges of slope-line approach lights. M. S. Gilbert and H. J. Cory Pearson.*

CORRESPONDENCE

RESONANCE TESTING AND FLUTTER CALCULATIONS

Mr. Plantin in his paper "The Technique of Resonance Testing and Flutter Calculations as Applied to Fighter Aircraft Design" (February 1952 JOURNAL), states that the greatest value of the frequency parameter ($2\pi fc/V$) for which energy can be extracted from an air stream is 1.7.

The work at the National Physical Laboratory to which reference is made in the paper was concerned with coupled oscillations in pitching and normal translation of a two-dimensional aerofoil. Contrary to the impression given by Mr. Plantin's statement, *vortex sheet theory for incompressible flow* leads to *no limitation* of the range of frequency parameters over which energy can be extracted from the stream. However, according to compressible flow theory, an upper limit to the frequency parameter for which energy can be extracted does appear at high subsonic speeds. This is the limit mentioned by Mr. Plantin. It is approximately 1.65 at $M=0.8$, 1.75 at $M=0.7$ and greater than 2.0 at $M=0.6$.

Complete information regarding binary oscillations involving flap rotation is not yet available, but for incompressible flow it has been shown theoretically that energy can be extracted from the stream up to a frequency parameter of 4.0 at least. Also for $M=0.7$ extrapolation suggests that it is possible to extract energy at frequency parameters above 2.0.

Experience gained from flutter speed calculations and model tests suggests that, for a practical system, the frequency of coupled flutter will most probably fall among the lower still-air resonance fequencies. Further, that the frequency parameter will probably be not greater than 1.5 for wing flutter, but it might be as high as 3.0 for control surface flutter. These values are based on experience; as yet there is no theoretical justification for a frequency-parameter limit that is universally applicable.

N. C. LAMBOURNE,
Aerodynamics Division,
National Physical Laboratory.

22*nd February* 1952.

I regret having associated the National Physical Laboratory with the figure of 1.7 for the limiting value of the frequency parameter for energy extraction from an air stream. The calculation was performed by Hawker Aircraft Ltd., using a method based on a National Physical Laboratory report.

The value is strictly applicable to a two-dimensional aerofoil performing coupled oscillations in an air stream at a Mach number of 0.7, the derivatives used being those for subsonic compressible flow theory.

The frequency parameter limit encountered in subsonic compressible flow was mentioned in my paper in order to show that there is some theoretical justification for the practice, based on experience, of limiting resonance tests to an upper frequency of approximately 50 c.p.s.

Our calculations were also extended to cover a typical tapered, swept-back fighter wing. The wing was considered to be oscillating according to arbitrary modes of parabolic flexure and linear torsion with respect to the flexural axis. The derivatives were those already used for the two-dimensional case appropriate to compressible flow at $M=0.7$. Treating the wing on a strip-theory basis, the limiting value of the frequency parameter for energy extraction from the air stream was found to be 0.67 (based on the tip chord). Subsequent binary flutter speed calculations using the same derivatives revealed a critical frequency parameter of 0.39. Both these values agree with experience and are much less than 1.7. This example at least would indicate that over-riding values are obtained for the limiting frequency parameter when two-dimensional aerofoils are considered.

C. P. PLANTIN, *Assoc. Fellow*,
Deputy Chief of Research and Development (Structures).
Hawker Aircraft Ltd.

11*th March* 1952.

APPOINTMENTS

This page of the JOURNAL is available for advertisements of appointments in the Industry, the Ministries, Universities and Colleges. The charge for advertisements for each inch, or part of an inch, in depth is £2 10s. 0d.

NEW ZEALAND.—Applications are invited from suitably qualified persons for appointment as **Flight Testing Officer,** Civil Aviation Branch, Wellington, Air Department, New Zealand. Commencing salary will be £830 N.Z. per annum rising to £890 N.Z. per annum plus £50 a year flying allowance. Applicants must have had an extensive experience in ab-initio flying training, preferably experience as an instructor at a flying instructor school and administrative experience in flying training. They must also either hold or qualify for at least a commercial pilot licence, an "A" category instructor rating and an instrument rating. Further details together with the necessary application forms can be obtained from: The High Commissioner for New Zealand, 415 Strand, London, W.C.2, mentioning this paper and quoting reference No. 3/47/30. Completed applications should be lodged **not later than 30th April 1952.**

CHIEF DESIGNER DRAUGHTSMAN required S.E. London Area with experience of design and development of aircraft instrument mechanism. Applicants should write stating age and salary expected, with details of experience to **Box J.S.984** at 191 Gresham House, E.C.2.

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